University of Swaziland

Supplementary Examination, 2012/2013

BSc II, Bass II, BEd II, B.Eng II

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1

1.1 Find the absolute maximum and absolute minimum values of the function

$$f(x) = x\sqrt{4 - x^2}$$

on the interval $[-1, 2]$. [7]

1.2 Let

$$f(x) = x^4 - 2x^2$$

1.2.1 Find the critical points of f. [3]

1.2.2 Determine the intervals where f is increasing and where f is decreasing. [4]

1.2.3 Determine all local extrema of f. [1]

1.2.4 Determine the intervals where f is concave up and where f is concave down. [4]

1.2.5 Find all inflection points of f. [1]

QUESTION 2

Evaluate each of the following limits.

2.1 $\lim_{t \to 0} \dfrac{t(1 - \cos t)}{t - \sin t}$ [5]

2.2 $\lim_{x \to 1^+} \left(\dfrac{1}{x - 1} - \dfrac{1}{\ln x} \right)$ [5]

2.3 $\lim_{x \to (\pi/2)^-} \left(x - \dfrac{\pi}{2} \right) \sec x$ [5]

2.4 $\lim_{x \to 0^+} x^{-1/\ln x}$ [5]

QUESTION 3

3.1 The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder? [10]

3.2 The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400 m tall building increasing when the angle of elevation of the sun is $\pi/6$ radians? [10]

QUESTION 4

4.1 The base of a solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are squares with bases running from the x-axis to the curve. Find the volume of the solid. [10]

4.2 The region bounded by the curve $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ is revolved about the line $y = 2$ to generate a solid. Find the volume of the solid. [10]

QUESTION 5

5.1 Find the length of the curve $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $1 \leq x \leq 2$ [10]

5.2 The region bounded by the lines $y = x$, $y = -x/2$ and $x = 2$ is revolved about the y-axis to generate a solid. Use the method of cylindrical shells to calculate the volume of the solid. [10]

QUESTION 6

6.1 Investigate the convergence of each series.

6.1.1 $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$ 6.1.2 $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$ [5,5]

6.2 Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}}$. [10]

QUESTION 7

7.1 Determine whether the sequence whose nth term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ is convergent or not. If it is convergent, find $\lim_{n\to\infty} a_n$. [10]

7.2 Consider the sequence $\{a_n\}$ defined recursively by

$$a_1 = 2 \qquad a_{n+1} = \tfrac{1}{2}(a_n + 6) \quad \text{for } n = 1, 2, 3, \ldots$$

7.2.1 Use mathematical induction to show that $a_{n+1} > a_n$ for all $n \geq 1$. [4]

7.2.2 Use mathematical induction to show that $a_n < 6$ for all n. [4]

7.2.3 Use your answers to 7.2.1 and 7.2.2 to determine whether or not the sequence is convergent. [2]

END OF EXAMINATION PAPER