UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2012/13

BSC./B.ED./B.A.S.S II

TITLE OF PAPER	:	CALCULUS II
COURSE NUMBER	:	M212
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ for $f(x, y, z) = 3x^2y - \sin(2yz^3)$ [6]

(b) Show that $f(x, y) = \cos(x - y)$ is a solution to $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = 0$ [4]

(c) Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ for

$$f(x, y) = x^4 - 4x^3y + 4xy^3 - y^4$$

[10]

QUESTION 2

(a) For each of the following use a double integral to find the area bounded by the curves

(i) $y = x^2$ and $y = x^4$

(ii) $y = 4x + 8$ and $y = x^3 + 8$ [12]

(b) A rectangular garden is to be fenced on 3 sides using 1000 metres of fencing (the 4th side being a straight river's edge). Use Langrange mulitpliers to find the dimensions that would give the largest possible area. [8]

QUESTION 3

(a) Find the critical points of the following functions and test for relative maxima, minima and saddle points

$$f(x,y) = 4xy - x^4 - y^4$$

[10]

(b) Show that the ellipsoid $3x^2 + 4y^2 + 8z^2 - 24 = 0$ and the hyperboloid of two piece $4x^2 - 4y^2 - z^2 - 4 = 0$ are orthogonal (perpendicular) to each other at the common point $p_o\left(4\frac{\sqrt{5}}{5}, \sqrt{2}, 2\frac{\sqrt{5}}{5}\right)$ [10]

QUESTION 4

a) Show that the function

$$f(x,y) = \frac{xy}{x-y}$$

satisfies

$$x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy} = 0$$

[10]

b) Evaluate the interated integral

$$\int_0^8 \int_{+3\sqrt{y}}^2 e^{x^4}\, dx dy$$

[10]

QUESTION 5

a) Express the given rectangular equations in polar

(i) $xy = 4$

(ii) $x^2 - 8x + y^2 + 7 = 0$ [4]

b) Consider the curve

$$r = 2 + 2\sin\theta.$$

(i) Sketch the curve.

(ii) Find the area enclosed by the curve.

(iii) Find the length of the curve. [16]

QUESTION 6

a) Find the equation of the tangent to the surface

$f(x, y) = x^2 + 3y^2 - 4z^2 + 3xy - 10yz + 4x - 5z - 22$

at the point $(1, -2, 1)$. [10]

b) Find the point on the plane

$$x + 2y - 3z - 4 = 0$$

nearest to the origin. [10]

QUESTION 7

a) Evaluate the integral by converting to polar coordinates

$$\int_o^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2 y^2 dx dy$$

[10]

b) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-y^2-x^2}} x^3 yz dz dy dx$$

[10]