UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2012/13

BSC./B.ED./B.A.S.S II

TITLE OF PAPER	:	CALCULUS II
COURSE NUMBER	:	M212
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Suppose that $z = f(x, y), x = r\cos\theta$ and $y = r\sin\theta$.

Prove that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2.$$

[10]

(b) Find the directional derivative of

$$z = f(x, y) = x^3 e^y + xz$$

in the direction of the vector from $P_1(4, 0, 16)$ to $P_2(-2, 1, 4)$. [10]

QUESTION 2

(a) Find the volume under the surface

$$z = x^4 y^4$$

and over the circle $x^2 + y^2 = 1$. [12]

(b) (i) Sketch the graph of the curve

$$r = 1 - \sin\theta.$$

(ii) Find the area of the region enclosed by the curve in (i). [8]

QUESTION 3

(a) Find the equation of the tangent surface $xyz^3 + yz^2 = 4$ at the point $(1, 2, 1)$.

(b) Find the equation of the plane through the 3 points $P(1, 2, 3), Q(-2, 0, 4)$ and $R(5, 2, -1)$.

(c) Evaluate

$$\int\int_R \frac{x}{\sqrt{x^2 + y^2}} dxdy$$

where R is the region bounded by the lines $y = x, y = -2$ and $x = 0$.

QUESTION 4

a) Given that

$$f(x, y) = x^2 + xy + y^2 \sin\left(\frac{x}{y}\right)$$

(i) Find f_x, f_y, f_{xx}, f_{xy} and f_{yy}.

(ii) Verify that

$$xf_x + yf_y = 2f$$

and that

$$x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy} = 2f.$$

[5,3,4]

b) Using a double integral, find the area of the region bounded by the curves $xy = 2, x = 2\sqrt{y}$ and $y = 4$. [8]

QUESTION 5

a) Find and classify the critical points of the function

$$f(x, y) = y^3 + x^2 - 6xy + 3x + 6y.$$

[10]

(b) Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y, z) = xyz$$

subject to

$$x^2 + y^2 + z^2 = 1.$$

[10]

QUESTION 6

a) Consider the cardioid

$$r = 1 - \cos\theta.$$

(i) Sketch the cardioid.

(ii) Find the length of the cardioid [12]

b) Find an equation in polar co-ordinates for each of the following curves

(i) $2x + 3y = 3$

(ii) $x^2 - 2x + y^2 = 0$ [8]

QUESTION 7

Evaluate the following integral

(a) $\int_0^1 \int_0^{\sqrt{x-x^2}} y^2 dydx$ [10]

(b) $\int_0^1 \int_0^{\sqrt{1-z^2}} \int_0^{\sqrt{1-y^2-x^2}} x^3yzdxdydz$ [10]