University of Swaziland

Final Examination May 2013

Title of Paper : Ordinary Differential Equations
Course Number : M213
Time Allowed : Three Hours

Instructions

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer **ANY FIVE** questions. Submit solutions to **ONLY FIVE** questions.
4. Show all the necessary steps.
5. A table of Laplace Transformations provided at the end of the question paper.

This paper should not be open until permission has been given by the invigilator.

Question 1

a) If $y = x$ is particular solution of the differential equation

$$y' = 4x^2(y-x)^2 + \frac{y}{x},$$

find its general solution.

Mark = 10pts

b) Solve the differential equation

$$(2x+3y-5)\frac{dy}{dx} + 3x + 2y - 5 = 0$$

Mark = 10pts

Question 2

a) Solve

$$(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x.$$

Mark = 10pts

b) Use Laplace transforms to solve

$$y'' + 2y' - 3y = 3, y(0) = 4, \; y'(0) = -7.$$

Mark = 10pts

Question 3

a) Solve

$$[y(\log x) - 1]y \; dx = x \; dy.$$

Mark = 10pts

b) Solve

$$xdx + ydy = \frac{a^2(xdy - ydx)}{x^2 + y^2},$$

where a is a constant.

Mark = 10pts

Question 4

a) Find the general solution of the the differential equation

$$y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}.$$

Mark = 12pts

b) Investigate the existence of solutions of the initial value problem

$$y' = \frac{1 + 2x + 3y}{2 + x^2 + y^2}, \; y(0) = 0,$$

over the rectangle R:$|x| \leq 2, |y| \leq 1$.

Mark = 8pts

Question 5

a) Solve

$$x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = \ln x$$

Mark = 12pts

b) It is given that $y_1 = x$ and $y_2 = \frac{1}{x}$ are two linearly independent solutions of the associated homogeneous equation of

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x, x \neq 0.$$

Find a particular integral and the general solution of the equation.

Mark = 8pts

Question 6

a) Solve

$$\begin{aligned}\frac{dx}{dt} + 4x + 3y &= t \\ \frac{dy}{dt} + 2x + 5y &= e^t\end{aligned}$$

Mark = 12pts

b) If

$$\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) / N = f(x)$$

a function of x alone, then show that $e^{\int f(x)dx}$ is an integrating factor of

$$M(x,y)dx + N(x,y)dy = 0.$$

Mark = 8pts

Question 7

Find the series solution, about $x = 0$, of the equation

$$xy'' + y' - xy = 0,$$

by the Frobenius method.

Mark = 20pts

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\cdot\left(ae^{at}-be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$