University of Swaziland

Supplementary Examination July 2013

Title of Paper : Ordinary Differential Equations
Course Number : M213
Time Allowed : Three Hours

Instruction

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions. Submit solutions to ONLY FIVE questions.
4. Show all the necessary steps.
5. A table of Laplace Transformations provided at the end of the question paper.

This paper should not be open until permission has been given by the invigilator.

Question 1

a) As you know the equation

$$y' + p(x)y = q(x)y^n$$

(with dependent variable y)is non linear for $n \neq 0, 1$. By changing the dependent variable reduce to a linear differential equation and find an integrating factor.

Mark = 10pts

b) Solve the differential equation

$$y(2xy + e^x)dx = e^x dy$$

.

Mark = 10pts

Question 2

a) If $y_1(x), y_2(x), ..., y_m(x)$ are m solutions of the linear homogeneous equation

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-}} + .. + a_{n-1}(x)\frac{dy}{dx} + a_n y = 0$$

on I, then show that a linear combinations $c_1y_1(x)+c_2y_2(x)+...+c_my_m(x)$, where $c_1, c_2, ..., c_m$ are constants is also a solution of the given differential equation on I.

Does this result hold for non-homogeneous equation?

Does this result hold for non-linear equation?

Mark = 10pts

b) Solve the differential equation

$$y'' + 5y' + 4y = x^2 + 7x + 9$$

Mark = 10pts

Question 3

a) Solve

$$(2x - 4y + 5)dy + (-x + 2y - 3)dx = 0$$

Mark = 14pts

b) Solve the following differential equation using two methods

$$(2x + e^y)dx + xe^y dy = 0$$

Mark = 6pts

Question 4

a) Use Laplace transforms to solve

$$y'' + 3y' + 2y = 3,\ \ y(0) = y'(0) = 1$$

Mark = 10pts

b) Find the general solution of the equation

$$y'' + 3y' + 2y = 2e^x$$

using the method of variation of parameters.

Mark = 10pts

Question 5

a) Solve

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = 2x^4$$

Mark = 14pts

b) Solve

$$ydx - xdy = xydx$$

Mark = 6pts

Question 6

a) Find the solution of the system of equation

$$3\frac{dy}{dt} + y + 3\frac{dx}{dt} = 3t + 1$$
$$\frac{dy}{dt} - 3y + \frac{dx}{dt} = 2t$$

Mark = 10pts

b) Find the general solution of the differential equation

$$y' = 3y^2 - (1 + 6x)y + 3x^2 + x + 1$$

if y=x is the solution of the differential equation.

Mark = 10pts

Question 7

Find a series solution about x = 0 of the equation

$$xy'' + y' + xy = 0$$

by the Frobenius method.

Mark = 20pts

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\dfrac{n}{s^{n+1}}$
$\dfrac{1}{\sqrt{t}}$	$\sqrt{\dfrac{\pi}{s}}$
e^{at}	$\dfrac{1}{s-a}$
$t^n e^{at}$	$\dfrac{n}{(s-a)^{n+1}}$
$\dfrac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\dfrac{1}{(s-a)(s-b)}$
$\dfrac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\dfrac{s}{(s-a)(s-b)}$
$\sin(at)$	$\dfrac{a}{s^2+a^2}$
$\cos(at)$	$\dfrac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\dfrac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\dfrac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\dfrac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\dfrac{a}{s^2-a^2}$
$\cosh(at)$	$\dfrac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\dfrac{2a^2}{s^4+4a^4}$