UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2012/13

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Does the point $R(4,4)$ lie on the line through $P(1,1)$ and $Q(2,2)$? [4]

(b) Find the angle between $\overline{a} = \overline{i} - \overline{j}$ and $\overline{b} = \overline{j} - \overline{k}$. [4]

(c) Use vector product to find the area of the parallelogram spanned by the vectors $\overline{a} = (2,3,-1)$ and $\overline{b} = (1,2,-4)$. [5]

(d) Find the volume of parallelepiped spanned by the directed segements $\overline{OA}, \overline{OB}$ and $\overline{OC}$, if the coordinates of A, B and C are $(2,2,2), (0,2,2), (2,0,8)$, respectively [7]

QUESTION 2

(a) If $f(x) = x^3 - x$, find all numbers in the interval $(0,1)$ for which the mean value theorem is satisfied. [3]

(b) Compute

(i) $\lim\limits_{x\to 0} \dfrac{\sin x - e^x + 1}{x^2}$,

(ii) $\lim\limits_{x+\infty} \dfrac{x^3 + 2x + 2}{2x^3 + x^2 - 3x + 1}$,

(iii) $\lim\limits_{x\to 0^+} (e^x - 1)\cot x$. [3,3,5]

(c) Use the quadratic approximation to compute $\sqrt{1+x}$ for small $|x|$ and estimate the error. In particular compute $\sqrt{18}$. [6]

QUESTION 3

(a) Find the fourth Taylor polynomial at $x_0 = 0$, for $f(x) = e^{-x}$. [4]

(b) Find the partial derivatives of $f(x,y) = x^2y^5$, at a point $P(-1,2)$. [3]

(c) Use the chain rule to evaluate f'_u and f'_v if $f(x,y) = x^2 - y^2, \quad x = u^2 + v^2, \quad y = 2uv$. [6]

(d) Verify equality of mixed derivatives theorem for

$$f(x,y) = \ln\sqrt{x^2 + y^2}.$$

[3]

(e) If $f(x,y) = 4\sin 2x \cos 2y$, what is df? [4]

QUESTION 4

a) For the function $f = x^3 - y^2 + yz - 12x$ find

(i) the gradient,

(ii) the stationary points. [2,2]

b) Find and classify all stationary points if $f(x, y) = x^3 + y^3 - 3x - 3y$. [4]

c) Use the method of Lagrange to find the extreme value of $f(x, y) = x^2 - y^2$, subject to constraint $2x + y = 5$. [5]

(d) Find three positive numbers whose sum is 24 and whose product is as large as possible. [7]

QUESTION 5

a) Compute the volume under the graph of $z = f(x, y) = x + 4y$ over the region $0 < x < 2$, $1 < y < 2$. [4]

b) Compute $\iint_D x^2 y\,dxdy$ if D is the interior of the triangle with vertices $(0,0), (0,1), (1,0)$. [5]

c) Use the separation of variables to evaluate integral of $f = e^{-x}\sin 2y$,

if $0 < x < 1, \quad 0 < y < \frac{\pi}{4}$. [4]

d) Compute $\iint_D (x^2 + y^2)^{\frac{3}{2}}\,dxdy$ in polar coordinates, if D is a region in the first quadrant bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, and the coordinate axes. [7]

QUESTION 6

a) Compute $\int\int_D\int (x^2 + y^2 + z^2)\ dx\ dy\ dz$, where D is a cube

$0 < x < 1,\ \ 0 < y < 1,\ \ 0 < z < 1.$ [6]

b) Pass to the spherical coordinates to evaluate $\int\int_D\int z^2 dx\ dy\ dz$, where D is the volume bounded by the sphere $x^2 + y^2 + z^2 = 4$. [8]

c) Separate the variables to solve the following initial value problem,
$y' = \frac{x^3}{y^2}$, $y = 2$ when $x = 0$. [6]

QUESTION 7

a) Prove that if the function $f(x, y)$ is homogeneous of degree 0, then $f(x, y)$ is function of $\frac{y}{x}$ alone. [3]

b) Consider the following ODE

$$3(3x^2 + y^2)dx - 2xydy = 0.$$

(i) Show that the coefficients are the homogeneous functions.

(ii) Solve the equation. [1,4]

c) Consider ODE

$$(3t^2 \sin^2 x)dt + (2t^3 \sin x \cos x - 2e^{2x})dx = 0.$$

(i) Test it for exactness,

(ii) Solve ODE.

[2,5]

(d) Solve the following initial value problem,

$$y'' - 2y' - 3y = 0,$$

$$y(0) = 4,\quad y'(0) = 0$$

[5]