UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2012/13

BSc./B.Ed./B.A.S.S II/B.ENG II

TITLE OF PAPER	:	LINEAR ALGEBRA
COURSE NUMBER	:	M220
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find conditions λ and u for which the following system of linear equations has

(i) a unique solution

(ii) no solution

(iii) infinitely many solutions.

$$\begin{aligned} x + y + z &= 0 \\ 2x + 3y + z &= 1 \\ 4x + 7y + \lambda z &= u \end{aligned}$$

[10]

(b) Let $T : R^2 \longrightarrow R^3$ be given by

$$T(x, y) = (k - 2y, 2x + y, x + y).$$

Find the matrix of T

(i) with respect to the standard basis

(ii) with respect to B^1 and B where

$B^1 = \{(1, -1), (0, 1)\}$ and $B = \{(1, 1, 0)(0, 1, 1,)(1, -1, 1)\}$ [10]

QUESTION 2

(a) Determine whether the following sets of vectors in the vector space $P_2(x)$ are linearly dependent. For those that are linearly dependent express the last vector as a linear combination of the rest

(i) $\{2x^2 + x, x^2 + 3, x\}$

(ii) $\{2x^2 + x + 1, 3x^2 + x - 5, x + 13\}$ [10]

(b) Let $S = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ be a set of non-zero vectors in a vector space V. Prove that S is linearly dependent if and only if one of the vectors V_j is linear combination of the proceeding vectors in S.

[10]

QUESTION 3

(a) Let V be the set of all ordered pairs of real numbers. Define addition and scaler multiplication as follows

$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1)$ and $\alpha(x, y) = (\alpha x_1 + \alpha - 1, \alpha y_1 + \alpha - 1)$

Show that V is a vector space [8]

(b) Find the inverse A^{-1} of the matrix A using the augmented matrix $[A : I]$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

(c) Use (b) to find a finite sequence of elementary matrices $E_1, E_2, \cdots E_k$ such that

$E_k E_{h-1} \cdots E_2 E_1 A = I$ [8]

QUESTION 4

(a) Show that the vector $(-3, 12, 12)$ is a linear combination of the vectors $(1, 0, 2)$ $(0, 2, 4)$ and $(-1, 3, 2)$ [8]

(b) Show that the set of vectors

$V = \{(0, 2, 1), (1, 0, 2), (1, -1, 0)\}$ is a basis for R^3

[8]

(c) Determine whether the following has a non trivial solution

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ 2x_1 + x_2 - x_3 + 2x_4 &= 0 \\ 3x_1 + 2x_2 + 2x_3 + 2x_4 &= 0 \end{aligned}$$

[4]

QUESTION 5

(a) Show that each eigen vector of a square matrix A is associated with only one eigenvalue

[5]

(b) Show that $A = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

is a skew symmetrix

[5]

(c) Find the characteristic polynomial eigen values and eigen vectors of the following matrix

$$A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$$

[10]

QUESTION 6

Let $B = \{u_1 u_2, u_3\}$ and $B^1 = \{\gamma_1, \gamma_2, \gamma_3\}$ be bases in R^3, where

$u_1 = (1,0,0) \quad u_2 = (1,1,0) \quad u_3(1,1,1)$

$\gamma_1 = (0,2,1) \quad \gamma_2(1,0,2) \quad u_3 = (1,-1,0)$

(a) Find the transition matrix from B^1 to B.

(b) Let $T : R^3 \to 3$ be the linear transformation whose matrix with respect to the basis B is

$$\begin{pmatrix} 3 & -6 & 9 \\ 0 & 3 & -6 \\ 0 & 0 & 0 \end{pmatrix}$$

Find the matrix of T w.r.t B^1 [8]

(c) If $\begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$ is the coordinates relative to B, find the co-ordinates relative to B^1 [4]

[6]

QUESTION 7

(a) Verify Cayley-Hamilton Theorem for the following matrix

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

[5]

(b) By inspection, find the inverses of the following elementary matrices

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ .4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[5]

(c) Use Crammer's rule to solve

$$\begin{aligned} 2x_1 + 8x_2 + x_3 &= 10 \\ 2x_3 + 3x_2 - x_1 &= -2 \\ 4x_1 + 4x_2 - 5x_3 &= 4 \end{aligned}$$

[5]

(d) Show that $2x^2 + 2x + 3$ is not a linear combination of

$x^2 + 2x + 1, \quad x^2 + 3, \quad x - 1$ [5]