UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2012/2013

B.Sc. / B.Ed. / B.A.S.S.II

TITLE OF PAPER	:	FOUNDATIONS OF MATHEMATICS
COURSE NUMBER	:	M231
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Criticize the following definitions.

i. A circle is a plane figure enclosed by one line. [2]

ii. An even integer is an integer that is twice an odd integer. [2]

iii. A quadratic polynomial is $2x^2 - 3x + 7$. [2]

iv. A rational number is a number that is not irrational. [2]

v. An algebraic number is 0. [2]

(b) In each of the following, state whether the statement is a proposition or not. Support your answer.

i. The square root of any integer is a non-negative real number. [2]

ii. If $x < 1$, then $x^2 < 1$. [2]

iii. The cube root of any integer is a real number. [2]

iv. For every angle t, $\sec^2 t - \tan^2 t = 1$. [2]

v. $x^2 + y^2 > 1$ (where x and y are real numbers). [2]

QUESTION 2

(a) Use the quantifiers *for all* or *there is* or their equivalents to make each of the following a true statement.

i. $(x-1)^2 = x^2 - 2x + 1$. [1]

ii. $|x| = x$. [1]

iii Not all triangles are isosceles. [1]

iv. No triangles are parallelograms. [1]

(b) Write the negation of the following definition.

> The real number u is a *least upper bound* for a set S of real numbers if and only if u is an upper bound for S and $\forall$ real numbers $\varepsilon > 0$, $\exists x \in S$ such that $x > u - \varepsilon$.

[6]

(c) A certain island has two kinds of inhabitants; knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described below. If you you cannot determine what these people are, can you draw any conclusions?

i. A says, "B is a knight," and B says, "The two of us are opposite types." [5]

ii. A says, "At least one of us is a knave," and B says nothing. [5]

QUESTION 3

(a) Translate each of the following statements into logical expressions using predicates (or statements involving variables), quantifiers and logical connectives.

i. No one is perfect. [2]

ii. Not everyone is perfect. [2]

(b) Express the statement

Some old dogs can learn new tricks

using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the words "It is not the case that.") [6]

(c) For each of the arguments below, explain which rules of inference are used at each step.

i. Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job. [3]

ii. Somebody in this class enjoys whale watching. Everybody who enjoys whale watching cares about ocean pollution. Therefore, there is someone in this class who cares about ocean pollution. [3]

(d) What is meant by a *sound argument*? Consider the following argument:

All rational numbers are integers. The real number 1 is a rational number. Therefore, the real number 1 is an integer.

Is the argument valid or invalid, and is it sound or unsound? Explain. [4]

QUESTION 4

(a) Determine whether each of the arguments below is valid or invalid. If an argument is valid, what rule of inference is used? If it is not, what logical error occurs?

i. If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$. [2]

ii. The number $\log_2 3$ is irrational if it is not the ratio of two integers. Therefore, since $\log_2 3$ cannot be written in the form a/b, where a and b are integers, then it is irrational. [3]

iii. If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then $n \leq 3$. [3]

iv. If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$. [2]

(b) Prove that $\sqrt{2}$ is irrational. [10]

QUESTION 5

(a) The temperature of a body is decreasing at 1% every second.

i. Find a formula for the temperature of the body at any time t. [8]

ii. If the initial temperature, at $t = 0$, is 300 °C, find the temperature of the body after 10 minutes. [3]

(b) Prove that there are infinitely many primes of the form $4k + 3$, where k is a nonnegative integer. [9]

QUESTION 6

(a) State and prove the Principle of Mathematical Induction I. [6]

(b) Find a formula for the sum

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)}.$$

Prove that your formula is correct. [8]

(c) Use the Principle of Mathematical Induction II to prove that the sum of the internal angles in an $n-$sided polygon is $(n-2)\pi$. [6]

QUESTION 7

(a) Express $1.813813813\ldots$ as a fraction $\frac{m}{n}$, where $m, n, \in \mathbb{Z}$ with $n \neq 0$. [5]

(b) Show that if $a_0.a_1a_2a_3\ldots$ and $b_0.b_1b_2b_3\ldots$ are two different decimal representations of the same real number, then one of them ends in $000\ldots$ and the other in $999\ldots$. [8]

(c) Prove that a real number is rational if and only if its decimal representation is repeating. [7]

END OF EXAMINATION