UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2012/2013

B.Sc. / B.Ed. / B.A.S.S.II

TITLE OF PAPER	:	FOUNDATIONS OF MATHEMATICS
COURSE NUMBER	:	M231
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Write each of the following statements in symbolic form.

i. John is either sick in the head or rich (or both). [2]

ii. If he is a sore loser, then he will not play with him again. [2]

iii. He either knows what he is doing or he is crazy (but not both). [2]

iv. All natural numbers are positive. [2]

v. That it is below freezing point is a necessary condition for it to be snowing. [2]

(b) Determine the conditions on the hypothesis and the conclusion under which each of the following statements is true or false, and give your reasons.

i. If $2 > 7$, then $1 > 3$. [1]

ii. If $2 < 7$, then $1 > 3$. [1]

iii. If $x = 3$, then $1 < 2$. [1]

iv. If $x = 3$, then $1 > 2$. [1]

v. If $1 + 1 = 3$, then $2 + 2 = 4$. [1]

vi. If $1 + 1 = 3$, then $2 + 2 = 5$. [1]

vii. If pigs can fly, then $1 + 1 = 3$. [1]

viii. If $1 + 1 = 3$, then pigs can fly. [1]

ix. If $1 + 1 = 2$, then pigs can fly. [1]

x. If $2 + 2 = 4$, then $1 + 2 = 3$. [1]

QUESTION 2

(a) Using truth table analysis, state whether the following argument is valid or invalid.

> If he were talented, then he would become famous. If he became famous, then he would also become wealthy. He became famous then wealthy. Therefore, he is talented.

[7]

(b) Write, symbolically, the negation of the statement

$$\forall\, x \in X\ \exists\, n_0 \in \mathbb{N}, \forall\, n > n_0\ \exists\, \varepsilon > 0, |f_n(x) - f(x)| < \varepsilon.$$

[5]

(c) Steve would like to determine the relative scores of three colleagues in the last M231 test using three facts. First he knows that all the three colleagues got different scores. Second, he knows that if Fred did not get the highest score of the three, then Janice did. Third, he knows that if Janice did not get the lowest score, then Maggie got the highest score. Is it possible to get the relative scores of Fred, Maggie and Janice from what Steve knows? If so, who got the highest score and who got the lowest amongst the three? [8]

<u>QUESTION 3</u>

(a) Translate each of the following statements into logical expressions using predicates (or statements involving variables), quantifiers and logical connectives.

i. All your friends are perfect. [2]

ii. Everyone is your friend and is perfect. [2]

(b) Express the statement

There is no one in this class who knows French and Russian

using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. [6]

(c) For each of the arguments below, explain which rules of inference are used at each step.

i. Each of the 62 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zakes, a student in this class, can use a word processing program. [3]

ii. Everyone in Maputo lives within 120 kilometers of the ocean. Someone who lives in Maputo has never seen the ocean. Therefore, someone who lives within 120 kilometers of the ocean has never seen the ocean. [3]

(d) Suppose we are given the following three facts:

- I will be admitted to Greatmath University only if I am clever.
- If I am clever, then I do not have to work hard.
- I have to work hard.

What can be deduced? Will I or will I not be admitted to Greatmath University? [4]

QUESTION 4

(a) Prove that if n is an integer such that $n^3 + 5$ is odd, then n is even using:

i. The contrapositive method. [6]

ii. Proof by contradiction. [4]

(b) Prove that if there are at least 6 people at a party, then either 3 of them knew each other before the party, or 3 of them were complete strangers before the party. [10]

QUESTION 5

(a) **(The Tower of Hanoi)**. Suppose n rings, with different outside diameters, are slipped onto an upright peg, the largest ring at the bottom, the second largest on top of it, and so on, so that the smallest ring is at the top, to form a pyramid. Two other upright pegs are placed sufficiently far apart. We wish to transfer all the rings, one at a time, to the second peg to form an identical pyramid. During the transfers, we are not permitted to place a larger ring on top of a smaller one (which necessitates the third peg). What is the smallest number of moves necessary to complete the transfer? [12]

(b) Let $x = 0.a_1a_2a_3\ldots$, where for $n = 1, 2, 3, \ldots$, the value of a_n is the number 0, or 1, or 2, or 3, which is the remainder when n is divided by 4. Is x rational? If so, express x as a fraction $\dfrac{m}{n}$, where m and n are integers with $n \neq 0$. [8]

QUESTION 6

(a) Prove that the square root of any prime number is irrational. [10]

(b) Critic Ivor Smallbrain is watching the classic film $11.\overline{9}$ *Angry Men.* But he is bored and starts wondering idly exactly which rational numbers $\frac{m}{n}$ have decimal expressions ending in $000\ldots$ (that is, ending in repeating zeros). He notices that this is the case if the denominator n is 2, 4, 5, 8, 10, or 16, and wonders if there is a simple general rule which tells us which rational numbers have this property.

Help Ivor by proving that a rational number $\frac{m}{n}$ (in its lowest terms) has a decimal expression ending in repeating zeros if an only if the denominator n is of the form 2^a5^b, where a and b are nonnegative integers. [10]

QUESTION 7

(a) Give the definition of a countable set. [3]

(b) Show that if A and B are countable sets, then $A \cup B$ is countable. [9]

(c) Prove that the set of all rational numbers is countable. [8]

END OF EXAMINATION