UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2012/2013

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find parametric equations for the line of intersection of the planes $2x+y+z=4$ and $3x-y+z=3$. [4]

(b) Compute the acute angle between the lines $3x-4y+7=0$ and $x+y+1=0$.[4]

(c) Find the distance between the plane $2x-2y+z=4$ and the point $S(1,2,3)$.[4]

(d) Prove that the vector $\mathbf{v} = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$ is perpendicular to the plane $Ax + By + Cz = D$, where $A \neq 0$, $B \neq 0$, and $C \neq 0$ are constants. [4]

(e) Find the component of the force $\mathbf{F} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ in the direction of the displacement $\overrightarrow{\mathbf{PQ}}$, where $P(3,2,0)$ and $Q(4,6,7)$ are points in space. What is the work done by the force $\mathbf{F}$ in moving an object over the displacement $\overrightarrow{\mathbf{PQ}}$?[4]

QUESTION 2

(a) Find the equation for the plane through $P(1,0,-1)$, $Q(0,2,0)$, and $R(1,2,3)$.[5]

(b) Show that the planes $5x+y-z=-3$ and $5x+y-7=6$ are parallel, and find the distance between them. [4]

(c) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in space. Prove that $\mathbf{u} \times \mathbf{v}$ is perpendicular to the plane containing $\mathbf{u}$ and $\mathbf{v}$. [4]

(d) Given that $\mathbf{F}(t) = e^{2t}\mathbf{u} + e^{3t}\mathbf{v}$, where $\mathbf{u}$ and $\mathbf{v}$ are constant vectors, show that $\mathbf{F}''(t) - 5\mathbf{F}'(t) + 6\mathbf{F}(t) = \mathbf{0}$. [4]

(e) If $\mathbf{u}''(t) = 6t\,\hat{\mathbf{i}} - 12t^2\,\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$, $\mathbf{u}'(0) = 7\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, and $\mathbf{u}(0) = 5\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, find $\mathbf{u}(t)$. [3]

QUESTION 3

(a) In spherical coordinates (ρ, ϕ, θ), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{r} = \rho \sin\phi \cos\theta \hat{\mathbf{i}} + \rho \sin\phi \sin\theta \hat{\mathbf{j}} + \rho \cos\phi \hat{\mathbf{k}}.$$

Find:

(i) $\hat{\rho}$; [2]

(ii) $\hat{\phi}$; [2]

(iii) $\hat{\theta}$; and [2]

(iv) the velocity vector $\mathbf{v}$ [3]

for any particle moving in this coordinate system.

(b) Let $a(t) = a_1(t)\hat{\mathbf{i}} + a_2(t)\hat{\mathbf{j}} + a_3(t)\hat{\mathbf{k}}$ be a differentiable vector function, and let $\phi(t)$ be a differentiable scalar function. Prove that

$$\frac{\mathrm{d}(\phi \mathrm{a})}{\mathrm{d}t} = \phi\frac{\mathrm{da}}{\mathrm{d}t} + \mathrm{a}\frac{\mathrm{d}\phi}{\mathrm{d}t}.$$

[4]

(c) If $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ and $r = |\mathbf{r}|$, show that:

(i) $\nabla r = \frac{\mathbf{r}}{r}$,

(ii) $\nabla^2(\log r) = \frac{1}{r^2}$. [3,4]

QUESTION 4

(a) A particle starts from rest and moves in a straight line with acceleration $(16-2v^2)$, where v is its speed. Show that the particle has terminal velocity $V = \sqrt{8}$, and find an expression for v in terms of the distance traveled. [6]

(b) A body of unit mass moving in a straight line is projected with speed u from a point at a distance d from the origin. It is acted upon by a force $\frac{k}{x}$, where k is a constant and x is the distance from the origin. Show that

$$x = de^{\frac{u^2-v^2}{2k}},$$

where v is the body's speed. [5]

(c) A particle drops from rest under gravity in a medium which exerts a resistive force of kv per unit mass, where k is a constant and v is the speed. Show that the terminal velocity is given by

$$V = \frac{g}{k}.$$

Also show that the speed v and the distance traveled x at any time t are given by

$$v = V\left(1 - e^{\frac{-gt}{V}}\right)$$

and

$$x = Vt - \left(\frac{V^2}{g}\right)\left(1 - e^{\frac{-gt}{V}}\right).$$

[9]

QUESTION 5

(a) Let $x(t) = c_1 \cos(\omega t + \phi_1)$ and $y(t) = c_2 \cos(\omega t + \phi_2)$ be harmonic functions in standard form with the same angular frequency ω. What do we mean by x leads y, and when does x lag y? [1]

(b) State whether x leads or lags y in each of the following:

(i) $x = 2\cos(2t + \frac{\pi}{4})$, $y = 3\cos(2t + \frac{9\pi}{2})$ [2]

(ii) $x = \cos(3t)$, $y = \sin(3t)$. [2]

(c) Express $A\cos(\omega t) + B\sin(\omega t)$ in the standard form $C\cos(\omega t + \phi)$ when $A = 3\frac{1}{2}$ and $B = -1$. [2]

(d) Find the current $I(t)$ in an RLC−circuit with $R = 100$ ohms, $L = 0.1$ henries, and $C = 10^{-3}$ farads, which is connected to a source of voltage $E(t) = 155\sin 377t$, assuming zero charge and current when $t = 0$. [13]

QUESTION 6

A projectile of mass m is launched with initial speed U at an angle θ with the horizontal. If the projectile has acting upon it a force due to air resistance equal to $-\beta\mathbf{v}$, where β is a positive constant and $\mathbf{v}$ is the instantaneous velocity, prove that the position at any time is given by

$$\mathbf{r} = \frac{mU}{\beta}(\cos\theta\mathbf{j} + \sin\theta\mathbf{k})(1 - e^{-\beta t/m}) - \frac{mg}{\beta}(t + \frac{m}{\beta}e^{-\beta t/m} - \frac{m}{\beta})\mathbf{k}.$$

[20]

QUESTION 7

(a) A particle of unit mass moves subject to a central force. Determine the law of force if the path followed by the particle is $r = a\cos\theta$, where a is a constant. [8]

(b) Suppose that a particle mass m is acted upon by a force $\alpha r^{-2} + \beta r^{-3}$ per unit mass (where ($\beta = \frac{1}{2}\alpha a$)) directed towards the origin $r = 0$ of an inertial frame. Suppose, also, that at $\theta = 0$ and $t = 0$, measurements of distance and velocity of the particle show that it is at distance a from the origin moving with velocity $\sqrt{\alpha/a}$ in a direction perpendicular to the radius vector. If $u = 1/r$, prove that

$$u = \frac{2}{a} - \frac{1}{a}\cos\frac{\theta}{\sqrt{2}}.$$

[12]

END OF EXAMINATION