UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2012/2013

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find unit vectors tangent and normal to the curve $f(x) = \sqrt{x^2+4}$ at the point $(-2, 2)$. [5]

(b) Determine whether the lines

$$L_1: \quad x = 3 + 2t, \quad y = 2 + t, \quad z = t; \qquad -\infty < t < \infty$$

and

$$L_2: \quad x = 1, \quad y = s, \quad z = -2 + s; \qquad -\infty < t < \infty$$

intersect, and if they do, find their point(s) of intersection. [4]

(c) Compute the acute angle between the lines $3x - 4y + 7 = 0$ and $x + y + 1 = 0$.[4]

(d) A girl is pulling a sled horizontally in a straight line over a distance of $100\,m$ by exerting a force of $50\,N$. The rope she is pulling on is at an angle of $45\,\text{deg}$ above the horizontal. How much work has she done? [4]

(e) Evaluate the limit $\lim_{t\to 1}\left(\frac{1}{t}\hat{\mathbf{i}} + \frac{\ln t}{t^2-1}\hat{\mathbf{j}} + \frac{t-1}{t^2-1}\hat{\mathbf{k}}\right)$. [3]

<u>QUESTION 2</u>

(a) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in space. Prove the *Pythagorean Principle,*

$$|\mathbf{u}+\mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 \iff \mathbf{u}\cdot\mathbf{v} = 0.$$

[5]

(b) Find the equation of the plane passing through the point $(0, 0, 2)$ and perpendicular to the vector $\hat{\mathbf{i}}$. [5]

(c) Find the angle between the planes $3x + 4y = 0$ and $2x + y - 2z = 5$. [4]

(d) Given the points $P(2, 1, -1)$, $Q(3, 0, 2)$, $R(4, -2, 1)$, and $S(5, -3, 0)$, find the volume of the parallelepiped having adjacent sides PQ, PR, and PS. [3]

(e) Given that $\mathbf{r}'(t) = [t\,e^{-t^2}, -e^{-t}, 1]$ and $\mathbf{r}(0) = [\frac{1}{2}, -1, 1]$, find $\mathbf{r}(t)$ at any time t. [3]

QUESTION 3

(a) the velocity vector $\mathbf{v}$ [1]

(b) the acceleration vector $\mathbf{a}$ [1]

(c) the speed [2]

(d) the unit tangent vector $\widehat{\mathbf{T}}$ [2]

(e) the curvature [5]

(f) the unit normal vector $\widehat{\mathbf{N}}$ [2]

(g) the unit binormal vector $\widehat{\mathbf{B}}$ [3]

(h) the tangential component of acceleration [2]

(i) the normal component of acceleration [2]

QUESTION 4

(a) A car with initial speed u accelerates uniformly over a distance of $2s$ which it covers in time t_1. It is then stopped by being retarded uniformly to rest over a distance s, which it covers in time t_2. Prove that

$$\frac{u}{2s} = \frac{2}{t_1} - \frac{1}{t_2}.$$

[4]

(b) A particle of unit mass is thrown vertically upwards with initial speed V. The air resistance at speed v is kv^2 per unit mass, where k is a constant.

(i) Show that H, the maximum height reached, is given by

$$H = \frac{1}{2k} \ln\left(\frac{g + kV^2}{g}\right),$$

and that the time T taken to reach this height is

$$T = \frac{1}{\sqrt{gk}} \tan^{-1}\left[\left(\frac{k}{g}\right)^{\frac{1}{2}} V\right].$$

[10]

(ii) Show that the particle return to the point of projection with speed v^*, where

$$v^* = V\left(\frac{g}{g + kV^2}\right)^{\frac{1}{2}}.$$

[6]

QUESTION 5

(a) From a point O, at height h above sea level, a particle is projected under gravity with a velocity of magnitude $\frac{3}{2}\sqrt{gh}$. Find the two possible angles of projection if the particle strikes the sea at horizontal distance $3h$ from 0. [10 marks]

(b) A particle is projected with velocity $\mathbf{u}$ from a point O in a vertical plane through the line of greatest slope of a plane inclined at an angle $-\beta$ to the horizontal. After time T, the particle strikes the inclined plane at the point P, at a distance R from O. If $\mathbf{u}$ makes an angle α with the horizontal, and if $|\mathbf{u}| = u$, show that:

(i) $T = \dfrac{2u\sin(\alpha+\beta)}{g\cos\beta}$ and $R = \dfrac{u^2[sin(2\alpha+\beta)+\sin\beta]}{g\cos^2\beta}$;

(ii) for constant u and β, R is maximum when $\alpha = \dfrac{\pi}{4} - \dfrac{\beta}{2}$. [8,2]

QUESTION 6

(a) The position of a particle moving along the x axis is determined by the equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 20\cos(2t).$$

If the particle starts from rest at $x = 0$, find

(i) x as a function of t,

(ii) the amplitude, period, and frequency after a long time. [7,3]

(b) The weight on a vibrating spring undergoes forced vibrations according to the equation

$$\frac{d^2x}{dt^2} + 4x = 8\sin(\omega t),$$

where x is the displacement from the equilibrium position and ω is a constant. If $x = 0$ and $\frac{dx}{dt} = 0$ when $t = 0$, find:

(i) x as a function of t,

(ii) the period of the external force for which resonance occurs. [8,2]

QUESTION 7

A comet moves in a plane under the gravitational attraction of the sun, which is situated at the origin O. Given that the attractive force between the sun and the comet can be written as

$$f(r) = -\frac{GMm}{r^2};$$

(a) Derive the equations

(i) $\ddot{r} - r\dot{\theta}^2) = -\frac{GM}{r^2};$ [6]

(ii) $r^2\dot{\theta} = h,$ [4]

where r and θ are plane polar coordinates, h is a constant, G is the gravitational constant, M is the mass of the sun, and m is the mass of the moon.

(b) Suppose that at the initial instant, $\theta = 0$, the comet is at distance d from the sun and is moving with speed v in a direction perpendicular to the radius vector from the sun. Show, by means of the substitution $r = \frac{1}{u}$, that the equation of the particle is

$$\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + u = \frac{GM}{d^2v^2}.$$

[10]

END OF EXAMINATION