University of Swaziland

Final Examination, 2012/2013

BSc III, Bass III, BEd III

Title of Paper : Numerical Analysis I

Course Number : M311

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1

1. (a) Convert the decimal 5.125 into its binary equivalent. [6 Marks]

   (b) Convert the binary $(0.\overline{101})_2$ into its decimal equivalent. [6 Marks]

   (c) Determine the machine representation in single precision on a 32-bit word length computer for the decimal number -12.75. [8 Marks]

QUESTION 2

2. Consider the equation

$$f(x) = x^3 + 4x^2 - 10 \tag{1}$$

   (a) Show that equation (1) has **exactly** one root in $[1, 2]$. [6 Marks]

   (b) By performing 4 iterations of the bisection method, show that this root lies in the interval $[1.3125, 1.375]$. [8 Marks]

   (c) How many iterations would be required to locate this root to a tolerance of 10^{-5}? [6 Marks]

QUESTION 3

3. (a) i. Interpolate the table

x	-0.5	0	0.5
$f(x)$	0.146	0.169	0.202

   using a suitable polynomial in Newton form. [8 marks]

   ii. Use your previous result to approximate $f(0.4)$. [2 marks]

   (b) Interpolate the table

x	0	3	-2	4
$f(x)$	1	181	-39	801

   with a suitable polynomial in Lagrange form. [10 marks]

QUESTION 4

4. (a) Suppose the table

x	0	h	$2h$
$f(x)$	$f(0)$	$f(h)$	$f(2h)$

is interpolated by a polynomial $P_2(x)$ of degree at most 2.

i. Write down the Lagrange representation of $P_2(x)$. [5 marks]

ii. Derive the numerical integration rule

$$\int_0^{3h} f(x)\,dx \approx \frac{3h}{4}\left[f(0) + 3f(2h)\right]$$

by integrating $P_2(x)$ between 0 and $3h$ [7 marks]

(b) Let $f(x) = \sqrt{9-x} - 3$

i. Re-write $f(x)$ in a new form $g(x)$ in such a way that loss of significance is avoided. [3 marks]

ii. Compare the results of calculating $f(0.0001)$ and $g(0.0001)$ using **five digits** and **chopping**. [3 marks]

QUESTION 5

5. (a) Find the coefficients below for the three-point Gaussian quadrature rule:

$$\int_{-1}^{1} f(x)dx \approx af\left(-\sqrt{\frac{3}{5}}\right) + bf(0) + cf\left(+\sqrt{\frac{3}{5}}\right)$$

[8 marks]

(b) Estimate $\int_{-3}^{3} \frac{1}{t^2+1}\,dt$ using this rule, and express your answer as a fraction. [12 marks]

QUESTION 6

6. (a) Find the LU factorisation of the matrix

$$A = \begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 2 & 2 & 0 \\ 0 & 2 & 8 & -6 \\ 0 & 0 & -6 & 10 \end{pmatrix}$$

where the diagonal elements of L are all 1s. [10 marks]

(b) Solve the linear system

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix}$$

using the LU factorisation obtained in (6a) [10 marks]

QUESTION 7

7. (a) Estimate the root of the equation

$$\ln x - e^x + 3 = 0$$

using 3 iterations of each of the following methods.

i. Newton method with starting point $x_0 = 1$. [6 marks]

ii. Secant method with starting points $x_0 = 1$ and $x_2 = 2$. [8 marks]

(b) Consider the bisection algorithm starting with the interval $[1.9, 2.1]$.

i. What is the width of the interval at the 9-th step of the iteration? [3 marks]

ii. What is the maximum distance possible between the true solution $x^\star$ and the mid-point x_9 of this interval? [3 marks]