University of Swaziland

Supplementary Examination, 2012/2013

BSc III, Bass III, BEd III

Title of Paper : Numerical Analysis I

Course Number : M311

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1

1. (a) Convert the decimal 115.25 into its binary equivalent. [6 Marks]

   (b) Convert the binary $(0.1\overline{10})_2$ into its decimal equivalent. [6 Marks]

   (c) Convert the single precision machine number

   $$[0\,10\,000\,011\;111\,010\,011\,000\,000\,000\,000\,00]_2$$

   to its decimal equivalent.. [8 Marks]

QUESTION 2

2. Consider the function

   $$f(x) = x - \cos x \tag{1}$$

   (a) Show that equation (1) has **exactly** one root in $[0, \pi/2]$. [6 Marks]

   (b) Determine the closed interval in which the root lies upon performing 4 iterations of the bisection method. [8 Marks]

   (c) How many iterations would be required to locate this root to a tolerance of 10^{-8}? [6 Marks]

QUESTION 3

3. (a) i. Interpolate the table

x	-0.2	0	0.2
$f(x)$	0.164	0.196	0.209

   using a suitable polynomial in Lagrange form. [8 marks]

   ii. Use your previous result to approximate $f(-0.1)$. [2 marks]

   (b) Interpolate the table

x	0	3	-2	4
$f(x)$	1	181	-39	801

   with a suitable polynomial in Newton form. [10 marks]

QUESTION 4

4. (a) Suppose the table

x	0	h	$2h$
$f(x)$	$f(0)$	$f(h)$	$f(2h)$

is interpolated by a polynomial $P_2(x)$ of degree at most 2.

i. Write down the Lagrange representation of $P_2(x)$. [5 marks]

ii. Derive the numerical integration rule

$$\int_0^{3h} f(x)\,dx \approx \frac{3h}{4}\left[f(0) + 3f(2h)\right]$$

by integrating $P_2(x)$ between 0 and $3h$ [7 marks]

(b) Find the roots of the following quadratic equation (as accurately as possible) using 8 digits and rounding

$$x^2 - 100000x + 1 = 0$$

[8 marks]

QUESTION 5

5. (a) Find the coefficients below for the three-point Gaussian quadrature rule:

$$\int_{-1}^{1} f(x)dx \approx af\,(-1) + bf(0) + cf\,(+1)$$

[8 marks]

(b) Estimate $\int_0^{\pi/4} \cos^2 t\,dt$ using this rule. [12 marks]

QUESTION 6

6. Solve the linear system

$$\begin{aligned} 4x_1 - 2x_2 &= 2 \\ -2x_1 + 2x_2 + 2x_3 &= 1 \\ 2x_2 + 8x_3 - 6x_3 &= 1 \\ -6x_3 + 10x_3 &= -1 \end{aligned}$$

using the LU factorisation. [20 marks]

QUESTION 7

7. (a) Estimate the root of the equation

$$x^3 - 2x + 2 = 0$$

using 3 iterations of the secant method with starting points $x_0 = 0$ and $x_2 = 1$. [8 marks]

(b) Evaluate $\int_0^2 \frac{x}{1+x}\,dx$ using Simpson's rule with $h = 0.5$. Find the error against the exact value of the integral to four decimal places. [6 marks]

(c) Consider the integral $\int_0^1 \sin\left(\frac{\pi x^2}{2}\right)\,dx$. Suppose we wish to integrate it numerically with an error of magnitude less than 10^{-5}. What width h is needed if we wish to use the composite Trapezoid rule? [6 marks]