UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2012/13

BSC./B.ED./B.A.S.S III

TITLE OF PAPER	:	ABSTRACT ALGEBRA I
COURSE NUMBER	:	M323
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find a prime factorization for each of the numbers: $a = 7200, b = 3132$. [,4]

(b) Use the factorization in (a) above to find $[a, b]$ and (a, b). [6]

(c) Find the number of generators of cyclic groups of order 12 and 42. [5]

(d) Solve the following system

$$\begin{aligned} 2x &\equiv 1 (mod\ 5) \\ 3x &\equiv 4 (mod\ 7) \end{aligned}$$

[5]

QUESTION 2

(a) Prove that a non-abelian group of order $2p, p$ prime, contains at least one element of order p. [8]

(b) Give a single numerical example to **disprove** the following:

"If $ka \equiv kb(mod\ n); a, b, k \in \mathbb{Z}$, then $a \equiv b(mod\ n)$". [4]

(c) Prove that every group of prime order is cyclic. [8]

QUESTION 3

(a) For $\mathbb{Z}_{12}$, find all subgroups and give a lattice diagram. [7]

(b)(i) Find all cosets of $H = \langle 6 \rangle$ in Z_{18}. [4]

(ii) Show that $\mathbb{Z}_6$ and S_3 are not isomorphic. [3]

(c) Find the number of elements in each of the cyclic subgroups

(i) $\langle 30 \rangle$ of $\mathbb{Z}_{12}$. [3]

(ii) $\langle 15 \rangle$ of $\mathbb{Z}_{48}$. [3]

QUESTION 4

(a) Given the definition of a group. What is an abelian group? [4]

(b) Show that the set $\mathbb{Q}$ with respect to the binary operation

$$a * b = a + b - 2013$$

is a group [8]

(c) Show that if G is a group, then the left and right cancellateion laws hold in G,

i.e. $ab = ac \Rightarrow b = c$, and $ba = ca \Rightarrow b = c$. [8]

QUESTION 5

(a) State Cayley's theoren [Do not prove]. [3]

(b) Define the notion of a normal subgroup of a group. [3]

(c) Consider $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$. Express the permutation as a product of

(i) disjoint cycles

(ii) transposition.

(d) Consider $\Pi = (1456), \sigma = (215)$ and $\rho = (16)(253)$ of S_6. Compute

(i) $\Pi\sigma$ (ii) $\sigma\Pi$ (iii) Π^2 and ρ^2. [8]

QUESTION 6

(a) Find $(616, 427)$ and express it in the form $616a + 427b$, where $a, b, \in \mathbb{Z}$. [5]

(b) Find all solutions of linear congruence

$$153x \equiv 6(mod\ \ 12)$$

[5]

(c) For any group G, show: If $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$, then G is abelian. [5]

(d) Prove that in any group, the identity element is unique. [5]

QUESTION 7

(a) Given groups G and H, define a group isomorphism ϕ from G to H. [4]

(b) Consider the mapping ϕ from $\mathbb{R}$ under addition to $\mathbb{R}^+$ under multiplication given by $(r)\phi = e^r$.

(i) Show that ϕ is a group isomorphism. [6]

(ii) What is ker ϕ?(i.e. the kernel of ϕ) [2]

(c) Let $H = \langle 3 \rangle$ be the subgroup of $\mathbb{Z}_{12}$ generated by the element $3 \in \mathbb{Z}_{12}$.

(i) Find all cosets of H in $\mathbb{Z}_{12}$. [4]

(ii) Give the group table for the quotient/factor group $\mathbb{Z}_{12}/H$. [4]