UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2012/13

B.Sc./ B.Ed. / B.A.S.S III

TITLE OF PAPER	:	DYNAMICS II
COURSE NUMBER	:	M355
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Give the definitions and some examples of

(i) degrees of freedom,

(ii) non-holonomic systems,

(iii) reonomic systems. [2,2,2]

(b) Prove that the frictional force is non-conservative. [3]

(c) Prove the cancellation of dot property lemma,

$$\frac{\partial \dot{\bar{r}}_\gamma}{\partial \dot{q}_i} = \frac{\partial \bar{r}_\gamma}{\partial q_i}.$$

[7]

(d) Consider a mathematical pendulum.

(i) Derive Lagrange equation, and

(ii) Solve it for small angle. [2,2]

QUESTION 2

a) Masses m_1 and m_2 are located on smooth inclined planes of fixed angles α_1 and α_2 respectivley and are connected by an inextensible string of negligible mass over a smooth peg. Set up the Lagrangian and find the acceleration of mass m_1. [5]

(b) The kinetic energy in spherical coordinates $r,\ \theta,\ \rho$ is given by

$2T = m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\rho}^2 \sin^2\theta)$ and the virtual work is given by

$\delta W = m(F\delta r + Gr\delta\theta + Hr\sin\theta\delta\rho)$.

(i) Find the generalized forces,

(ii) Construct Lagrange equations. [3,5]

(c) Prove that for holonomic, scleronomic system

$$\sum_{i=1}^{n} \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T,$$

in the usual notations. [7]

QUESTION 3

(a) Derive Hamilton's equations if $H = H(q,\ p,\ t)$. [5]

(b) For the particle of mass m in the field of gravity find

(i) generalized momentum,

(ii) Hamiltonian,

(iii) Hamilton's equations. [2,2,2]

(c) Let θ and ρ be generalized coordinates. Given kinetic energy

$2T = ml^2(\dot{\theta}^2 + \dot{\rho}^2 \sin^2\theta)$ and potential energy $\Pi = -mgl\cos\theta$. Find

(i) Generalized momenta,

(ii) Hamiltonian,

(iii) Hamilton's equations. [3,3,3]

QUESTION 4

a) Show that if $H = H(q,\ p)$ and a system is conservative then

$$H = T + \Pi$$

[4]

b) Consider a tranformation for the mathematical pendelum $x = l\sin\rho$.

(i) Show that $p_x = \dfrac{p_\rho}{l\cos\rho}$

(ii) Prove that the transformation $(\rho,\ \ p_\rho) \to (x, p_x)$ is canonical. [5,3]

(c) Let H be a Hamiltonian and D a dynamic variable of a system.

(i) Show that

$$\frac{dD}{dt} = \frac{\partial D}{\partial t} + [D, H].$$

(ii) Given $H = \dfrac{p^2}{2} - \dfrac{1}{2q^2}$. Show that $D = \dfrac{pq}{2} - Ht$ is a constant of motion. [3,5]

QUESTION 5

a) Prove that

$$[q_k, q_l]_{q,p} = [p_k, p_l]_{q,p} = 0$$

in the usual notations. [6]

b) Consider transformation

$$Q = q^\alpha e^{\beta p}, \quad P = q^\alpha e^{-\beta p},$$

where α and β are constants. Use Poisson Brackets to find α and β such that transformation is canonical. [6]

c) Derive Hamilton's equations in Poisson formulation. [8]

QUESTION 6

a) State and prove that the Main Lemma of calculus or variations. [6]

b) Find the extremals for the functional

$$V[y(x)] = \int_x^{x_1} (y^2 + y'^2 - 2y \sin x) dx.$$

[6]

c) Let $F = y\sqrt{1 - y'^2}$. Construct

(i) Euler equation,

(ii) Beltrami identity. [4,4]

QUESTION 7

a) Find extremals for the following functionals

(i) $V[y(x), z(x)] = \int_0^1 (y'^2 + z'^2 + y'z')dx;$

$y(0) = z(0) = 0, \quad y(1) = 1, \quad z(1)$, is free

(ii) $V[y(x)] = \int_0^1 (y''^2 + 1)dx;$

$y(0) = 0, \quad y'(0) = y(1) = y'(1) = 1.$ [6,8]

b) Find Ostrogradski's equation for the following functional

$$V[z(x,y)] = \int\int_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy,$$

where $z(x, y)$ is known on the boundary of region D. [6]