University of Swaziland

Supplementary Examination, July 2013

BSc IV, Bass IV, BEd IV, BEng III

Title of Paper : Partial Differential Equations

Course Number : M415

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions. **Submit solutions to ONLY FIVE questions.**
4. Show all your working.
5. A Table of Laplace Transforms is provided at the end of the question paper.

Question 1

(a) By eliminating the arbitrary function, find the partial differential equation satisfied by the following function

$$x + y + u = f(x^2 + y^2 + u^2).$$

[10]

(b) Find the general solution of the partial differential equation

$$x(x^2 + 3y^2)u_x - y(3x^2 + y^2)u_y = 2u(y^2 - x^2).$$

[10]

Question 2

Consider the following partial differential equation

$$2u_{xx} - 4uxy + 2u_{yy} + 3u = 0.$$

(a) Classify the partial differential equation as hyperbolic, parabolic or elliptic. [3]

(b) Reduce the equation into its canonical form and hence find the general solution. [17]

Question 3

Find the particular solutions for the following partial differential equations

(a) $yu_y - x^2u_y, \quad u = x^2$ on $3y^2 = 2x^3$. [10]

(b) $u_{xy} = 1, \quad u = 0$ and $u_x = 0$ on $x + y = 0$. [10]

Question 4

Consider the function

$$f(x) = \begin{cases} -1, & -\pi \leq x < 0; \\ 0, & x = 0; \\ +1, & 0 < x \leq \pi. \end{cases} \qquad f(x + 2\pi) = f(x).$$

(a) Find the fourier series expansion. [12]

(b) Use Parseval's identity to find the value of the sum

$$\sum_{n-1}^{\infty} \frac{1}{(2n-1)^2}$$

[8]

Question 5

Solve the following partial differential equations using the method of Laplace transforms

(a) $u_{xt} + \sin t = 0, \quad u(x,0) = x,\ u(0,t) = 0.$ [10]

(b) $xu_x + u_t = xt, \quad u(x,0) = 0,\ u(0,t) = 0.$ [10]

Question 6

Show that the initial value problem with non-homogeneous boundary conditions

$$\begin{aligned}
&u_t - u_{xx} = 0, \quad 0 < x < L, \quad t > 0\\
&u(0,t) = T_1, \quad t \geq 0\\
&u(L,t) = T_2, \quad t \geq 0\\
&u(x,0) = f(x), \quad 0 \leq x \leq 1
\end{aligned}$$

can be transformed to an initial value problem with homogeneous boundary conditions. Hence find the general solution of initial value problem using separation of variables. [20]

Question 7

Solve the following non homogeneous partial differential equation

$$\begin{aligned}
&u_{tt} - c^2 u_{xx} = e^x, \quad -\infty < x < +\infty, \quad t > 0\\
&u(x,0) = 2\\
&u_t(x,0) = x^2.
\end{aligned}$$

[20]

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{d^n f}{dt^n}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$