UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2012/13

BSC IV

TITLE OF PAPER	:	FLUID DYNAMICS
COURSE NUMBER	:	M455
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

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USEFUL FORMULAE

The gradient of a function $\psi(r, \theta, z)$ in cylindrical coordinates is

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{\partial\psi}{\partial z}\hat{k}$$

The divergence and curl of the vector field

$$\underline{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{k}$$

in cylindrical coordinates are

$$\nabla\cdot\underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\}$$

and

$$\nabla\times\underline{v} = \frac{1}{r}\det\begin{bmatrix}\hat{r} & r\hat{\theta} & \hat{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ v_r & rv_\theta & v_z\end{bmatrix}$$

The divergence of a vector

$$\underline{v} = v_r\hat{r} + v_\lambda\hat{\lambda} + v_\theta\hat{\theta}$$

in spherical coordinates

$$\nabla\cdot\underline{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial\lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta}$$

The convective derivative and Laplacian in cylindrical coordinates are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial\theta} + v_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

Identities

$$\underline{v}\cdot\nabla\underline{v} = \nabla\left(\frac{v^2}{2}\right) - \underline{v}\times\underline{\omega}$$

$$\nabla\times(\nabla\times\underline{a}) = \nabla\nabla\cdot\underline{a} - \nabla^2\underline{a}$$

QUESTION 1

(a) Define a particle path. [4]

(b) Describe the Eulerian method of treating motion of continuous medium. [4]

(c) A velocity field is specified as $\overline{V} = Axy\overline{i} + By^2\overline{j}$, where $A = 1m^{-1}s^{-1}$,

$B = -\frac{1}{2}\ m^{-1}\ s^{-1}$, and the coordinates are measured in meters.

(i) Find the dimension of flow fluid. Explane.

(ii) Calculate the velocity components at the point $(1, 2, 0)$.

(iii) Develop an equation for the streamline passing through this point [1,1,3]

(d) (i) Derive the formular for convective derivative of the density.

(ii) What is a physical meaning of convective derivative of velocity? [5,2]

QUESTION 2

(a) The three components of velocity are given by $u = Ax + By + Cz$,

$V = Dx + Ey + Fz$ and $w = Gx + Hy + Jz$.

Determine the relationship among the coefficients A through J that is necessary if this is to be a possible incompressible flow. [3]

(b) Consider two-dimensional incompressbile flow.

(i) Define the stream function $\psi(x, y)$,

(ii) Show that $\psi(x, y) = \int_b^y u(x, \eta)d\eta - \int_a^x U(\xi, b)d\xi$

where a and b are constants. [1,4]

(c) Determine the family of stream functions that will yield the velocity field

$\overline{V} = (x^2 - y^2)\overline{i} - 2xy\overline{j}$. [5]

(d) Incompressible flow around a circular cylinder of radius a has a stream function

$\psi(r, \theta) = -Ur\sin\theta + \dfrac{Ua^2\sin\theta}{r}$, U is constant.

(i) obtain an expression for the velocity field,

(ii) find V_r along the circle $r = a$,

(iii) Locate the points along $r = a$ where $|\overline{V}| = U$. [3,2,2]

<u>QUESTION 3</u>

(a) Consider a piston-cylinder apparatus. At one instant when the piston is L_0 away from the closed end of the cylinder the gas density is uniform at $\rho = \rho_0$, and the piston begins to move away from the closed end at $V = V_0$. The gas velocity is one-dimensional and proportional to the distance from the closed end; it varies linearly from zero at the end to $U = V_0$ at the piston.

(i) Show that $\frac{d\rho}{dt}|_{t=0} = -\rho_0 \frac{V_0}{L_0}$,

(ii) Find $\rho(t)$.

HINT: Density ρ is independent of x. [5,5]

(b) Water flows along a pipe whose area of cross-section $A(x)$ varies slowly with the coordinates x along the pipe. Flow is almost in x-direction.

(i) Explain average velocity over cross-section

(ii) What is Eulerian acceleration?

(iii) Show that Lagrange acceleration is $a_L = -\frac{Q^2}{A^3}\frac{dA}{dx}$, where Q is a volume flow rate. [2,1,4]

(c) Prove $\overline{V} = \nabla + \psi\overline{k}$ in the usual notations. [3]

QUESTION 4

a) Consider a so-called "'rigid-body rotation"' model: $V_r = 0, \quad V_\theta = Ar$, where A is a constant. Find stream function. [3]

b) Consider the Rankine's vortex

$$\varpi = \begin{cases} \Omega \overline{k}, & for \quad r < a \\ 0, & for \quad r > a \end{cases}$$

(i) Find the stream function,

(ii) Find velocity V_θ. [7,5]

HINT: $\nabla + (\nabla + \overline{A}) = \nabla\nabla \cdot \overline{A} - \nabla^2 \overline{A}$.

(c) A flow is respresented by the velocity field

$\overline{V} = (4x^2 + 3y)\overline{i} + (3x - 2y)\overline{j}$

Determine if the field is

(i) a possible incompressible flow,

(ii) irrational. [2,3]

QUESTION 5

a) Describe the stress components σ_n and τ_n. [4]

b) Consider liquid of constant density ρ in the field of gravity. Show that

$$p = p_0 - \rho g z$$

in the usual notations. [5]

c) Define the Newtonian fluid. [2]

d) The velocity distribution for laminar flow between fixed parallel plates is given by

$$u = u_{max}\left[1 - \left(\frac{2y}{h}\right)^2\right],$$

where h is the distance separating the plates, and the origin is placed midway between the plates. Consider $\mu = 1.1 \times 10^{-3}$kg/ms, $u_{max} = 0.05m/s$, $h = 5mm$. Calculate

(i) the shear stress on the lower plate and give direction,

(ii) the force on a $0.3m^2$ section of the lower plate and give its direction [4,2]

e) Write Navier-Stokes equation for inviscid flow. [3]

QUESTION 6

a) A velocity field in a fluid with density of $1500 kg/m^3$ is given by

$$\overline{V} = (Ax - By)t\bar{i} - (Ay + Bx)t\bar{j},$$

where $A = 1s^{-2}$, $B = 2s^{-2}$, x and y are meters, and t in seconds. Body and viscous forces are negligible.

(i) Find the acceleration of a field particle at point $(x, y) = (1, 2)$ and $t = 1 sec$.

(ii) Find the pressure gradient at the same point and same time. [6,2]

b) Consider steady viscous incompressible flow between two stationery plates located at $y = 0$ and $y = 1$. Given that pressure at $x = 0$ and $x = L$ is p_o and p_L respectively, with $p_0 > p_L$. The effect of body forces is negligible. Put $\overline{V} = u(x, y)\bar{i}$ and show that

(i) $\dfrac{\partial p}{\partial x} = \mu \dfrac{\partial^2 u}{\partial y^2}$,

(ii) $p(x) = p_0 - \dfrac{p_0 - p_L}{L} x$,

(iii) $u(y) = y(1 - y)\dfrac{p_0 - p_L}{2\mu L}$. [5,4,3]

QUESTION 7

a) Derive the Navier-Stokes equation in dimensionless form introducing the characteristic length and velocity. [6]

b) (i) Define Reynolds number,

(ii) Find dimension of Reynolds number. [2,2]

c) Water flow in a circular pipe. At one section the diameter is 0.3m, the static pressure is 260kpa (gage), the velocity is 3m/s, and the elevation is 10m above ground level. At a section downstream at ground level the pipe diameter is 0.15m. Friction effects may be neglected.

(i) Write Bernoulli's equation,

(ii) Find velocity at downstream section,

(iii) Calculate the gage pressure at the downstream section. [2,3,7]