University of Swaziland

Final Examination – November 2013

BSc I, BEng I, BEd I

Title of Paper : Algebra, Trigonometry & Analytic Geometry
Course Number : M111
Time Allowed : Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 3 (out of 5) questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Give a concise definition of each of the following terms.

i. The *minor* of a matrix [2 marks]

ii. The *conjugate* of a complex number [2 marks]

b. State

i. de Moivre's Theorem [2 marks]

ii. The Remainder Theorem [2 marks]

c. Sketch the graph of $y = 1 - e^x$. [2 marks]

A.2 a. Work out

i. $\left(x - \frac{2}{x}\right)^5$ (using the *Binomial theorem*) [5 marks]

ii. $\frac{x^4 - x^2 + x + 1}{x + 1}$ (using the *synthetic division*) [4 marks]

iii. $\frac{20i}{1 - 3i}$ (and express your answer in the form $x + iy$) [3 marks]

b. Given the vectors $\underline{A} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\underline{B} = 7\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$, find

i. $\underline{A} \cdot \underline{B}$ [2 marks]

ii. $\underline{A} \times \underline{B}$ [5 marks]

c. Find the *centre* and *radius* of the circle defined by

$$x^2 + y^2 - 14x + 10y - 70 = 0.$$ [4 marks]

d. Given that $\cos\theta = \frac{1}{\sqrt{5}}$ where $180^0 < \theta < 360^0$, find the *exact* value of $\sin\theta$. [4 marks]

e. Find the value of the sum

$$\sum_{n=0}^{\infty} 50\left(\tfrac{4}{9}\right)^n.$$ [3 marks]

Section B

Answer ANY 3 Questions in this section

B.1 a. Evaluate and express in the form $x + iy$.

i. $4i^{19}(2+3i) - 3i(4-2i)$ [4 marks]

ii. $2(\cos 240^0 + i\sin 240^0) \cdot 7(\cos 120^0 + i\sin 120^0)$ [2 marks]

b. Use de Moivre's thoerem to evaluate (and express in the form $x + iy$)

$$\left(1 - i\sqrt{3}\right)^6.$$ [4 marks]

c. Consider the polynomial

$$P(z) = 6z^4 + Az^3 + Bz^2 + 4z - 8,$$

where both A and B are real coefficients.

i. Given that $z = 2i$ is a root of $P(z)$, find the values of A and B. [4 marks]

ii. Hence find all the other roots of $P(z)$. [6 marks]

B.2 a. Find the exact value of $\cos 832\frac{1}{2}^0$. [3 marks]

b. Given that $\sin A = \frac{\sqrt{7}}{4}$, $\cos B = \frac{3}{5}$ where A is in QII while B is in QIV, find the exact values of

i. $\sin(A+B)$ [2 marks]

ii. $\cos(A+B)$ [2 marks]

Hence state, with reasons, the qudrant in which the angle $A + B$ lies. [2 marks]

c. Prove

$$\cos(A+B)\cos(A-B) = 1 - \sin^2 A - \sin^2 B.$$ [6 marks]

d. Find the general solution of

$$2\cos^2\theta - \sin\theta - 1 = 0.$$ [5 marks]

B.3 a. Find the *first 4 terms* in the binomial expansion of

$$\left(\frac{1}{x^2} - 2x\right)^{\frac{3}{2}}.$$

[6 marks]

b. Find the term involving $\frac{1}{x^2}$ in the binomial xpansion of

$$\left(x^3 - \frac{2y}{x}\right)^{18}.$$

[6 marks]

c. The sum of the first n terms of a geometric progression (GP) with first term T_1 and common ratio r, is given by

$$S_n = T_1 \frac{(1 - r^n)}{1 - r}, \quad n \geqslant 1, \; r \neq 1.$$

i. Derive this formula [3 marks]

ii. Hence find the sum of the *last 10 terms* in the GP

$$5, \; 10, \; 20, \; \cdots, 81\,920, \; 163\,840.$$

[5 marks]

B.4

a. Express as a single logarithm with coefficient 1, and simplify

$$4 - 4\log_2\left(2\sqrt{a}\right) + 2\log_2\left(a\sqrt{2}\right).$$

[5 marks]

b. Solve

i. $7^{2x-1} = 2$ [3 marks]

ii. $30e^{x} + 8e^{-x} = 53$ [5 marks]

c. Consider the formula

$$y = \ln t - \ln(kt + 1).$$

i. Make t the subject of the formula. [5 marks]

ii. Find the value of y (correct to 3 s.f.) if $k = 7$ and $t = 40$. [2 marks]

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B.5

a. Given that $A = \begin{pmatrix} 4 & -1 & 1 \\ 2 & 0 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix}$, work out

$$AA^T - BB^T.$$ [5 marks]

b. Find the value of the determinant

$$\begin{vmatrix} 1 & 0 & -1 \\ \sec\theta & \sin\theta & -\cos\theta \\ -\csc\theta & \cos\theta & \sin\theta \end{vmatrix}.$$ [5 marks]

c. Use mathematical induction to prove the formula

$$T_1 + T_1 r + T_1 r^2 + T_1 r^3 + \cdots + T_1 r^{n-1} = T_1 \frac{(1-r^n)}{1-r}, \quad n \geqslant 1,\ r \neq 1.$$ [10 marks]