University of Swaziland

6

Supplementary Examination – July 2014

BSc I, BEng I, BEd I

Title of Paper	:	Algebra, Trigonometry & Analytic Geometry
Course Number	:	M111
Time Allowed	:	Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 3 (out of 5) questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Give a concise definition of each of the following terms.

i. The *minor* of a matrix [2 marks]

ii. The *conjugate* of a complex number [2 marks]

b. State two *double-angle* formulas for $\cos 2\theta$ [2 marks]

c. Sketch the graphs of $y = \sin\theta$ and $y = \cos\theta$ on the same axis in the interval $-180^0 \leqslant \theta \leqslant 180^0$. [2 marks]

A.2 a. Work out

i. $\left(2x^2 - 3\right)^5$ (using the *Binomial theorem*) [5 marks]

ii. $\dfrac{x^4 - 16}{x + 2}$ (using the *synthetic division*) [3 marks]

iii. $(1 + 2i)(1 - 3i)(1 + 4i)$ (and express your answer in the form $x + iy$) [3 marks]

iv. $\log_b b^5 + \ln e^{2m-5} - \log 10^{2-3m}$ [3 marks]

b. Given the vectors $\underline{A} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\underline{B} = 2\hat{\imath} + 2\hat{\jmath} - 1\hat{k}$, find

i. $\underline{A} \cdot \underline{B}$ [2 marks]

ii. $\underline{A} \times \underline{B}$ [5 marks]

c. Find the *centre* and *radius* of the circle defined by

$$x^2 + y^2 + 6y - 70 = 0.$$ [4 marks]

d. Given that $\sin\theta = \frac{1}{2}$ where $90^0 < \theta < 270^0$, find the *exact* value of $\sin\theta$. [3 marks]

e. Find the value of the sum

$$\sum_{n=-10}^{50} (18 - 7n).$$ [3 marks]

Section B

Answer ANY 3 Questions in this section

B.1 a. Evaluate and express in the form $x + iy$.

i. $\left(3 - 5i^{11}\right)^2$ [4 marks]

ii. $\dfrac{12(\cos 240^0 + i \sin 240^0)}{4(\cos 150^0 + i \sin 150^0)}$ [2 marks]

b. Use de Moivre's thoerem to evaluate (and express in the form $x + iy$)

$$\left(3 + 3i\right)^6.$$ [4 marks]

c. Find all the fourth roots of 81. [10 marks]

B.2 a. Find the exact value of $\cos 105^0$. [3 marks]

b. Given that $\cos A = \frac{\sqrt{3}}{2}$ where A is in QIV, find the exact values of

i. $\sin A$ [2 marks]

ii. $\cos 2A$ [4 marks]

c. Prove

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$$ [5 marks]

d. Find the general solution of

$$2\cos^2\theta - \sin\theta - 2 = 0.$$ [5 marks]

B.3 a. Find the *first 4 terms* in the binomial expansion of

$$\left(\frac{1}{x} - 2x\right)^{-2}.$$ [6 marks]

b. Find the term that does not involve x in the binomial expansion of

$$\left(x^4 + \frac{y}{x}\right)^{20}.$$ [6 marks]

c. Find 3 numbers in arithmetic progression whose product is 210 while their sum is 30. [8 marks]

B.4

a. Express as a single logarithm with coefficient 1, and simplify

$$2\ln\left(\frac{b^2c}{\sqrt{a}}\right) + 4\ln\left(\frac{a}{b\sqrt{c}}\right).$$ [5 marks]

b. Solve

i. $7^{3x-2} = 2$ [3 marks]

ii. $4^{x+1} - 5 \cdot 2^x = -1$ [5 marks]

c. The population of a city grows according to the formula

$$P(t) = 250,000e^{0.03t}$$

where t is time (in years) from the year 2000.

i. Find the population in the year 2014. [2 marks]

ii. Find the date when the population will be double that in 2000. [5 marks]

B.5

a. Given that $A = \begin{pmatrix} 4 & -1 & 1 \\ 2 & 0 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix}$, work out

$$\left(AA^T\right)\left(BB^T\right).$$

[5 marks]

b. Find the value of the determinant

$$\begin{vmatrix} 1 & 0 & -1 & 2 \\ 2 & 0 & -2 & 1 \\ 1 & -1 & 0 & -3 \\ -4 & 0 & 0 & 5 \end{vmatrix}.$$

[5 marks]

c. Use mathematical induction to prove that

$$P(n) = 3^{2n} - 1,$$

is always divisible by 4, where $n \geqslant 1$ is an integer. [10 marks]