University of Swaziland

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Final Examination – May 2014

BSc I, BEd I, BEng I, BASS I

Title of Paper	: Introduction to Calculus
Course Number	: M115
Time Allowed	: Three (3) hours

Instructions:

1. This paper consists of 2 sections.

2. Answer ALL questions in Section A.

3. Answer ANY THREE (3) questions in Section B.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. If direct substitution yields the indeterminate form $\frac{0}{0}$, how do we proceed in evaluating the limit

$$\lim_{x \to a} \frac{p(x)}{q(x)}?$$

[2 marks]

b. Evaluate

i. $\displaystyle \lim_{x \to -2} \left(\frac{7 - 2x^2}{4 - 2x - x^2} \right)$ [2 marks]

ii. $\displaystyle \lim_{x \to \frac{1}{2}} \left(\frac{6x^2 - x - 1}{1 - 4x^2} \right)$ [4 marks]

iii. $\displaystyle \lim_{x \to \infty} \left(\frac{4 + 2x - 9x^2}{6x^2 - x - 1} \right)$ [3 marks]

c. Sketch the graph of $y = 3H(x + 5)$ where $H(x)$ denotes the Heaviside function. [2 marks]

A.2 Find y' if

a. $y = 4x^3 - 6\sqrt{x} + \dfrac{3}{x^2}$ [2 marks]

b. $y = x \sin x + \cos x$ [3 marks]

c. $y = 4e^{2x} - 3e^{-4x}$ [2 marks]

d. $y = \ln \left(x^2 + 4x + 4 \right)$ [3 marks]

A.3 a. State the *Fundamental Theorem of Calculus*. [3 marks]

b. Integrate

i. $\displaystyle \int_1^4 \left(4x - \frac{7}{\sqrt{x}} + 1 \right) dx$ [4 marks]

ii. $\displaystyle \int \left(\sec^2 3\theta - e^{-2\theta} \right) d\theta$ [2 marks]

iii. $\displaystyle \int 8xe^{-x^2} dx$ [4 marks]

iv. $\displaystyle \int 8x^2 e^{-2x} dx$ [4 marks]

Section B

Answer ANY THREE (3) Questions in this section

B.4 a. Find the value of the limit

$$\lim_{x \to 0} \frac{4 - (3x + 8)^{\frac{2}{3}}}{x}$$ [6 marks]

b. Find y' and simplify

    i. $y = \dfrac{x + 3}{\sqrt{x^2 + 6x + 10}}$ [6 marks]

    ii. $y = \arctan \sinh 2x$ [3 marks]

c. Use Leibnitz rule to work out

$$\frac{\mathrm{d}^4}{\mathrm{d}x^4}\left(x^3 \ln x\right).$$ [5 marks]

B.5 Consider the function

$$y = 3x^4 + 12x^3 + 2.$$

a. Find the equation of the *normal* to the curve of y when $x = -1$ and express it in general form. [5 marks]

b. Find the *stationary point(s)* of the function and determine its(their) nature. [6 marks]

c. Find the *inflexion point(s)*. [4 marks]

d. Make a sketch of the graph. [5 marks]

B.6 Evaluate

a. $\displaystyle\int_0^3 \frac{x^2}{9+x^2}\,dx$ [10 marks]

b. $\displaystyle\int \frac{2x+1}{x^3-4x^2+4x}\,dx$ [10 marks]

B.7 a. Integrate

$$\int e^{-2x}\sin x\,dx.$$ [10 marks]

b. Find the *exact* value of the area of the region bounded by the parabola $y = x^2$ and the straight line $y = 2x+1$. [10 marks]

B.8 a. A company needs to design a closed rectangular box with a square base and a capacity of 12,000 cm³. If the base is to be made out of heavy-duty material which costs twice as much as the material for the sides and top, find the dimensions of such a box that will cost the least. [7 marks]

b. Use the *limit definition* to find $f'(x)$ given

$$f(x) = 1 - 4\sqrt{x}.$$ [7 marks]

c. Given $x^2 + y^2 = a^2$, where a is a constant, show that

$$y''y^3 + a^2 = 0.$$ [10 marks]