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**Supplementary Examination – July 2014**

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**BSc I, BEd I, BEng I, BASS I**

**Title of Paper** : Introduction to Calculus

**Course Number** : M115

**Time Allowed** : Three (3) hours

**Instructions:**

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY THREE (3) questions in Section B.
4. Show all your working.

**THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.**

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**Section A**  
**Answer ALL Questions in this section**

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**A.1** a. If direct substitution gives  $\frac{1}{0}$ , how do we proceed in evaluating the limit

$$\lim_{x \rightarrow a} \frac{p(x)}{q(x)}? \quad [2 \text{ marks}]$$

b. Evaluate

i.  $\lim_{x \rightarrow -3} \left( \frac{3 - 2x}{3 - x^2} \right)$  [2 marks]

ii.  $\lim_{x \rightarrow 2} \left( \frac{x^2 - x - 2}{4 - x^2} \right)$  [4 marks]

iii.  $\lim_{x \rightarrow \infty} \left( \frac{4 - 3x + x^2}{\frac{1}{2}x^2 + 2x - 4} \right)$  [3 marks]

c. Sketch the graph of  $y = 3H(x - 2)$  where  $H(x)$  denotes the Heaviside function. [2 marks]

**A.2** Find  $y'$  if

a.  $y = 4 - x^2 - \frac{2}{x}$  [2 marks]

b.  $y = (x - 1)e^x$  [3 marks]

c.  $y = 4 \sinh 2x - 3 \sin 4x$  [2 marks]

d.  $y = \ln(4 - 3x)$  [3 marks]

**A.3** a. State the *Fundamental Theorem of Calculus*. [3 marks]

b. Integrate

i.  $\int_{-1}^1 \left( 3 - \frac{7}{x^2} + 3x^2 \right) dx$  [4 marks]

ii.  $\int (\sin 2\theta - e^{-2\theta}) d\theta$  [2 marks]

iii.  $\int \frac{2x}{1 + x^2} dx$  [4 marks]

iv.  $\int xe^x dx$  [4 marks]

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**Section B**

**Answer ANY THREE (3) Questions in this section**

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**B.4** a. Find the value of the limit

$$\lim_{x \rightarrow 0} \frac{4 - \sqrt{3x + 16}}{x} \quad [6 \text{ marks}]$$

b. Find  $y'$  and simplify

i.  $y = \frac{x}{\sqrt{x^2 + 4}}$  [6 marks]

ii.  $y = \ln \left( \frac{1 + x^2}{1 - x^2} \right)$  [3 marks]

c. Use Leibnitz rule to work out

$$\frac{d^4}{dx^4} (x^3 \ln x). \quad [5 \text{ marks}]$$

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**B.5** Consider the function

$$y = x^4 - 4x^3 + 4x^2 - 5.$$

a. Find the equation of the *normal* to the curve of  $y$  when  $x = -1$  and express it in general form. [5 marks]

b. Find the *stationary point(s)* of the function and determine its(their) nature. [6 marks]

c. Find the *inflexion point(s)*. [4 marks]

d. Make a sketch of the graph. [4 marks]

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**B.6 Evaluate**

a.  $\int_0^2 \sqrt{4-x^2} dx$  [10 marks]

b.  $\int \frac{x+7}{x^2-2x^3} dx$  [10 marks]

**B.7 a. Integrate**

$$\int \frac{dx}{x^{\frac{1}{2}} + x^{\frac{2}{3}}}. \quad [10 \text{ marks}]$$

b. Find the *exact* value of the area of the region bounded by the parabola  $y = x^2$  and the straight line  $y = 4 - x^2$ . [10 marks]

**B.8 a.** Find the dimensions of the largest right circular cylinder that can be inscribed inside a right circular cone of height 20 cm and radius 6 cm.

[10 marks]

b. Use the *limit definition* to find  $f'(x)$  given

$$f(x) = 1 - \frac{1}{x}. \quad [10 \text{ marks}]$$