UNIVERSITY OF SWAZILAND

FINAL EXAMINATION, 2013/2014

B.Sc. II, BASS II, B.Ed II, B.Eng II

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION 1

1.1. Define each of the following:

(a) a *critical number* of a function f, (2)

(b) a *point of inflection* of a function f. (2)

1.2. Let $f(x) = x^{4/5}(x-4)^2$. Find the critical numbers of f. (4)

1.3 State the Increasing/Decreasing Test. (3)

1.4 True or False: If $f'(c) = 0$, then f has a local maximum or a local minimum at $x = c$. Explain your answer. (3)

1.5 What does l'Hopital's Rule say? Explain how you can use it if you have a product $f(x)g(x)$ where $f(x) \to 0$ and $g(x) \to \infty$ as $x \to a$. (5)

1.6 The region bounded by the curves $y = \sqrt{x}$ and $y = x^2$ is rotated about the line $y = 2$ to generate a solid. Set up, but do not evaluate, the integral for the volume of the solid. (5)

1.7 (a) Explain what it means to say that a sequence $\{a_n\}$ is convergent. (2)

(b) Explain what it means to say that a series $\sum_{n=1}^{\infty} a_n$ is convergent. (3)

1.8. True or False: If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent. Give reasons for your answer. (4)

1.9. State the Integral Test. (5)

1.10. What is meant by the radius of convergence of a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$? (2)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION 2 [*20 Marks*]

2.1. Consider the function $f(x) = x\sqrt{4 - x^2}$.

2.1.1. What is the domain of f? (2)

2.1.2. Find the absolute maximum and minimum values of f. (6)

2.2. Consider the function $f(x) = x^4 + 4x^3$.

2.2.1. Find the intervals on which f is increasing or decreasing. (4)

2.2.2. Find the local maximum and local minimum values of f. (2)

2.2.3. Find the intervals on which f is concave up and concave down. (4)

2.2.4. Find the inflection points of f. (2)

QUESTION 3 [*20 Marks*]

Evaluate the following limits.

3.1. $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$ (5)

3.2. $\lim_{x \to \infty} x^3 e^{-x^2}$ (5)

3.3. $\lim_{x \to 0} \left(\csc x - \cot x \right)$ (5)

3.4. $\lim_{x \to 0} (1 - 2x)^{1/x}$ (5)

QUESTION 4 [*20 Marks*]

4.1. The base of a solid is a circular disk with radius 3. Parallel cross-sections perpendicular to the base are isosceles right triangles with hypotenuse lying along the base. Find the volume of the solid. (10)

4.2. Use the method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by the curve $y = x^2$ and the lines $y = 0$, $x = 1$, and $x = 2$, about the line $x = 1$. (10)

QUESTION 5 [*20 Marks*]

5.1. Find the length of each of the following curves:

5.1.1. $x = 1 + 3t^2,\ y = 4 + 2t^3,\ 0 \leq t \leq 1$ (6)

5.1.2. $x = y^{3/2},\ 0 \leq y \leq 1$ (6)

5.2 The arc of the parabola $y = x^2$ from $(1, 1)$ to $(2, 4)$ is rotated about the y-axis. Find the area of the resulting surface. (8)

QUESTION 6 [*20 Marks*]

6.1 Consider the sequence $\{a_n\}$ defined by the recursive relation

$$a_1 = \sqrt{2}, \quad a_{n+1} = \sqrt{2 + a_n}.$$

6.1.1. Show that the sequence is increasing and bounded above by 3. (8)

6.1.2. Apply an appropriate theorem to determine whether or not the sequence is convergent. (2)

6.2 Find the radius of convergence and interval of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$. (10)

END OF EXAMINATION PAPER