University of Swaziland 23

Supplementary Examination, 2013/2014

B.Sc. II, BASS II, B.Ed II, B.Eng II

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION 1

1.1. True or False: If $f'(c) = 0$, then f has a local maximum or a local minimum at c. Explain your answer. (3)

1.2. Find the inflection points of $f(x) = x + \sqrt{1-x}$ (5)

1.3. (a) State l'Hopital's Rule. (3)

(b) Explain how you would use l'Hopital's Rule if you have a power $\left[f(x)\right]^{g(x)}$ where $f(x) \to 0$ and $g(x) \to 0$ as $x \to a$. (3)

1.4. Explain how you would use the closed interval method to find the absolute minimum and absolute maximum values of a function f on the interval $[a, b]$. (4)

1.5. Set up, but do not evaluate, an integral for the area of the surface obtained by rotating the curve $y = \ln x,\ 1 \leq x \leq 3$ about the x-axis. (5)

1.6. State the Monotonic Sequence Theorem. (2)

1.7. Given a series $\sum_{n=1}^{\infty} a_n$, let s_n denote its nth partial sum; $s_n = \sum_{i=1}^{n} a_i$. Explain what it means for the series to be convergent and for it to be divergent. (5)

1.8. State the Test for Divergence. (2)

1.9. Consider the following series:

$$\sum_{n=1}^{\infty} n^b \quad \text{and} \quad \sum_{n=1}^{\infty} b^n.$$

(a) Each series has a special name. What are these special names? (2)

(b) For what values of b does the series $\sum_{n=1}^{\infty} n^b$ converge? (2)

(c) For what values of b does the series $\sum_{n=1}^{\infty} b^n$ converge? (2)

1.10. What is meant by the radius of convergence of a power series? (2)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION 2 [*20 Marks*]

2.1. Find the absolute maximum and absolute minimum values of

$$f(x) = \ln(x^2 + x + 1)$$

on the interval $[-1, 1]$. (8)

2.2. Consider the function $f(x) = x^4 - 2x^2$.

2.2.1. Find the intervals on which f is increasing or decreasing. (4)

2.2.2. Find the local maximum and local minimum values of f. (2)

2.2.3. Find the intervals on which f is concave up and concave down. (4)

2.2.4. Find the inflection points of f. (2)

QUESTION 3 [*20 Marks*]

3.1. Evaluate the following limits.

3.1.1. $\displaystyle\lim_{x\to 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$ (5)

3.1.2. $\displaystyle\lim_{x\to 0} \left(\cot x - \frac{1}{x} \right)$ (7)

3.2. Consider the sequence $\{a_n\}$ where $a_n = 2n \sin \frac{\pi}{n}$. Show that

$$\lim_{n\to\infty} a_n = 2\pi.$$

(8)

QUESTION 4 [*20 Marks*]

4.1. The base of a solid is an elliptical region with boundary curve $9x^2 + 4y^2 = 36$. Parallel cross-sections perpendicular to the x-axis are isosceles right triangles with hypotenuse lying along the base. Find the volume of the solid. (10)

4.2. Use the method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by the curve $y = x^3$ and the lines $y = 0$, $x = 1$, and $x = 2$, about the line $x = 1$. (10)

QUESTION 5 [*20 Marks*]

5.1. Find the length of each of the following curves:

5.1.1. $x = e^t - t,\ y = 4e^{t/2},\ -8 \le t \le 3.$ (6)

5.1.2. $y = \dfrac{(x^2+2)^{3/2}}{3},\ 0 \le x \le 3.$ (6)

5.2 The curve $y = \sqrt{4 - x^2}$, for $-1 \le x \le 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the x-axis. (8)

QUESTION 6 [*20 Marks*]

6.1 Consider the sequence $\{a_n\}$ defined by the recursive relation

$$a_1 = 2, \quad a_{n+1} = \frac{1}{2}(a_n + 6).$$

6.1.1. Show that the sequence is increasing and bounded above by 6. (8)

6.1.2. Apply an appropriate theorem to determine whether or not the sequence is convergent. (2)

6.2 Find the radius of convergence and interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$. (10)

END OF EXAMINATION PAPER