UNIVERSITY OF SWAZILAND 27

FINAL EXAMINATION, 2013/2014

B.Sc. II, B.Eng II, B.Ed II, BASS II

Title of Paper : Calculus II

Course Number : M212

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked**.
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

28

SECTION A [*40 Marks*]: Answer ALL Questions

A1. (a) For each of the following use a double integral to find the area bounded by the curves

(i) $y = x^3 + 8, \quad y = 4x + 8$

(ii) $x = y(y-2), \quad y + x = 12$ (10)

(b) Show that the function

$f(x,y) = \frac{xy}{x-y}$

satisfies

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = 0$$ (10)

A2. (a) Find and classify the critical points of

$f(x,y) = 4xy - x^4 - y^4$ (10)

(b) Evaluate

$$\int_0^1 \int_0^{+\sqrt{x-x^2}} y^2 dy dx$$ (10)

SECTION B: Answer any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) Use the Lagrange multipliers to find the maximum and minimum values of the function

$$f(x,y) = 3x + 4y$$

subject to the constraint

$$x^2 + y^2 = 1$$

(10)

(b) Find the volume under the surface $f(x,y) = e^{-(x+y)}$ and above the region of the $xy-$ plane bounded by $y = x, x = \frac{1}{2}, x = 1$ and $y = 0$ (8)

(c) Express in polar form

$$x^2 + y^2 = \sqrt{x^2 + y^2} - 4x$$

(2)

29

QUESTION B2 [*20 Marks*]

B2. (a) (i) Sketch the graph of the curve

$$v = 1 - \cos\theta$$

(ii) Find the length of the curve (6)

(b) Use the chain rule to find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial s}$ where (4,4)

$$z = x^2 \sin y, \quad x = r^2 + s^2, \quad y = 2rs.$$

(6)

(c) Evaluate

$$\int_0^1 \int_0^{x^2} \int_{xy}^{x+y} xyz\,dz\,dy\,dx$$

(8)

QUESTION B3 [*20 Marks*]

B3. (a) Assuming that the equation

$$x^3 + x^2y - x^2z + z^3 - 2 = 0$$

defines z implicitly as a function of x and y, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(b) Find $\frac{dz}{dt}$ if $z = \frac{x^2 - y^2}{x^2 + y^2}$, $x = t^2 - 3t + 2 \quad y = -t^2 - 5t + 7$ (4)

(c) Evaluate the iterated integral

$$\int_0^8 \int_{3\sqrt{y}}^2 e^{x^4}\,dx\,dy.$$

(8)

QUESTION B4 [*20 Marks*]

B4. (a) Evaluate the iterated integral by first converting to polar co-ordinates.

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2y^2\,dx\,dy.$$

(8)

(b) Find the directional derivative of the function

$$f(x, y, z) = x^3e^y + xz$$

30

at the point (4016) in the direction of the vector $v = 6i - \hat{j} + 12\hat{h}$ (8)

(c) Express in rectangular form

$$r^2 = 9\sin 2\theta$$

(4)

QUESTION B5 [*20 Marks*]

B5. (a) Sketch the cardinal

$$r = 1 + \sin\theta$$

(5)

(b) Find the area inside the cardiod in (a) (3)

(c) Evaluate the following integrals

(i) $\int\int_v\int (x^2 + y^2)dxdydz$

where v is described by $1 \le x \le 2,\quad 0 \le y \le 1 \quad 2 \le z \le 5$

(ii) $\int\int_v\int (2x - y - z)dxdydz$

v is described by $0 \le x \le 1,\quad 0 \le y \le x^2 \quad 0 \le z \le x + y$ (7)

END OF EXAMINATION PAPER