UNIVERSITY OF SWAZILAND 31

SUPPLEMENTARY EXAMINATION, 2013/2014

B.Sc. II, B.Eng II, B.Ed II, BASS II

Title of Paper : Calculus II

Course Number : M212

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.

   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked**.
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: Answer ALL Questions

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A1. (a) Given that

$$f(x,y) = x^2 + xy - y^2 \sin(\frac{x}{y})$$

(5)

(i) Find f_x, f_y, f_{xx}, f_{xy} and f_{yy}. (5)

(ii) Verify that

$$xf_x + yf_y = 2f$$

and that (3)

$$x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy} = 2f.$$

(4)

(b) Using a double integral, find the area of the region bounded by the curves $xy = 2, x = 2\sqrt{2}$ and $y = 4$. (8)

A2. (a) Find and classify the critical points of the function

$$f(x,y) = y^3 + x^2 - 8xy + 3x + 6y.$$

(b) Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x,y,z) = xyz$$

subject to

$$x^2 + y^2 + z^2 = 1.$$

(10)

SECTION B: Answer any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) Consider the cardioid

$$r = 1 - \cos\theta$$

(i) Sketch the cardioid.

(ii) Find the length of the cardioid (10)

(b) Find an equation in polar co-ordinates for each of the following curves

(i) $2x + 3y = 3$

(ii) $x^2 - 2x + y^2 = 0$ (12)

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QUESTION B2 [*20 Marks*]

B2. (a) Evaluate the following integral

(a) $\int_0^1 \int_0^{\sqrt{x-x^2}} y^2 dy dx$ (8)

(b) $\int_0^1 \int_0^{\sqrt{1-y^2-x^2}} x^3 yz dx dy dz$ (10)

QUESTION B3 [*20 Marks*]

B3. (a) Suppose that $z = f(x, y), x = r\cos\theta$ and $y = r\sin\theta$.
Prove that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2$$

(10)

(b) Find the directional derivative of

$$z = f(x, y) = x^3 e^y + xz$$

in the direction of the vector from $P_1(4, 0, 16)$ to $P_2(-2, 1, 4)$. (10)

QUESTION B4 [*20 Marks*]

B4. (a) Find the volume under the surface

$$z = x^4 y^4$$

and over the circle $x^2 + y^2 = 1$. (10)

(b) (i) Sketch the graph of the curve

$$f = 1 - \sin\theta.$$

(ii) Find the area of the region enclosed by the curve in (i). (12)

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QUESTION B5 [*20 Marks*]

B5. (a) Find the equation of the tangent surface $xyz^3 + yz^2 = 4$ at the point $(1, 2, 1)$. (8)

(b) Find the equation of the plane through the 3 points $P(1, 2, 3)$. $(Q(-2, 0, 4)$ and $(5, 2, -1)$. (5)

(c) Evaluate

$$\int\int_R \frac{x}{\sqrt{x^2 + y^2}} dx dy.$$

where R is the region bounded by the lines $y = x$, $y = -2$ and $x = 0$. (3)

END OF EXAMINATION PAPER