University of Swaziland

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Supplementary Examination, July 2014

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Ordinary Differential Equations

Course Code : M213

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) Define each of the following terms and give an example

  (i) Boundary value problem. [3]

  (ii) Homogeneous function. [3]

  (iii) Linear ordinary differential equation. [3]

  (iv) Regular singular point for a second order linear ordinary differential equation. [3]

(b) Given that if $y_1(x)$ and $y_2(x)$ are two different solutions of

$$\frac{dy}{dx} + P(x)y = Q(x).$$

Is the function $r(x) = c_1(y_1(x) - y_2(x)) + c_2 y_2(x)$ a solution of the differential equation where c_1 and c_2 are constants? Explain. [3]

(c) By eliminating the constants find the differential equation satisfied by the following functions

  (i) $y = a\cos(2x) + b\sin(2x)$. [7]

  (ii) $y = ax + a - a^3$. [5]

(d) Show that

$$\mathcal{L}[cos(at)] = \frac{s}{s^2 + a^2}.$$

[5]

(e) Given the differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)y^n,$$

show that the transformation

$$v = y^{1-n}$$

transforms the equation into a first order linear differential equation

$$\frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x).$$

[8]

SECTION B: ANSWER ANY 3 QUESTIONS

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Question 2

(a) Using the substitution $u = \ln x$. Find the general solution of

$$x^2y'' - xy' + y = 0$$

[8]

(b) Solve the following differential equation

$$y'' + 2y' + y = \frac{e^{-x}}{x}.$$

Use the method of variation of parameters to find the particular solution. [12]

Question 3

Solve the following differential equations

(a) $y' + 3x^2y = xe^{-x^3}$. [5]

(b) $y' = \dfrac{xy + x^2}{x^2}$. [10]

(c) $y'' + \dfrac{5}{x}y' + \dfrac{4}{x^2}y = 0$. [5]

Question 4

(a) Show that if $y_1(x)$ is any particular solution of

$$\frac{dy}{dx} = P(x)y^2 + Q(x)y + R(x)$$

then the transformation

$$y = y_1(x) + \frac{1}{v(x)}$$

transforms the equation into a first order linear differential equation

$$\frac{dv}{dx} + (2P(x)y_1(x) + Q(x))v = -P(x).$$

Hence solve the equation

$$\frac{dy}{dx} = -y^2 + 2xy - x^2 + 1, \quad y_1 = x.$$

[13]

(b) Solve the following differential equation 43

$$(y^2 - 4)dx - (x^2 - 1)dy = 0.$$

[7]

Question 5

(a) Given that the differential equation

$$\mu(x)M(x,y)dx + \mu(x)N(x,y)dy = 0$$

is exact. Show that

$$\mu(x) = \exp\left(\int \frac{M_y - N_x}{N} dx\right)$$

[8]

(b) Find the second solution of

$$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 3y = 0, \quad x \neq 0$$

given that $y_1 = x^{-1}$ is a solution. Show that this solution, along with the given solution form a fundamental set of solutions for the differential equations. [12]

Question 6

Given the differential equation

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + ky = 0$$

(a) find two linearly independent solutions of the form $\sum_{n=0}^{\infty} a_n x^n$. [17]

(b) Show that if k is an even integer, then one of the solutions in (a) is a polynomial of degree n. [3]

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Table 1: Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}[f(t)]$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at} - e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at} - be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{d^n f}{dt^n}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$g(t) = \begin{cases} 0, & 0 \le t \le a; \\ f(t-a), & a < t. \end{cases}$	$e^{-as}F(s)$