UNIVERSITY OF SWAZILAND

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SUPPLEMENTARY EXAMINATIONS 2013/2014

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	FOUNDATIONS OF MATHEMATICS
COURSE NUMBER	:	M231
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ALL QUESTIONS IN SECTION A. 3. ANSWER ANY THREE QUESTIONS IN SECTION B.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

SECTION A

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QUESTION 1

Define the following;

(a) Proposition; [2]

(b) Hypothesis; [2]

(c) Logical equivalence; [2]

(d) Premiss; [2]

(e) Deductive reasoning; [2]

(f) Inductive reasoning; [2]

(g) Sound argument; [2]

(h) Formal fallacy; [2]

(i) Circular reasoning; [2]

(j) Counter example. [2]

QUESTION 2

(a) Show that $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$ is a tautology. [8]

(b) Use truth table analysis to show that:

(i) $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$; [6]

(ii) $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$. [6]

SECTION B

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QUESTION 1

(a) Determine the following sets:

(i) $\{n \in \mathbb{N} : \exists m \in \mathbb{N} \text{ with } m \leq n\}$; [3]

(ii) $\{n \in \mathbb{N} : \forall m \in \mathbb{N} \text{ we have } m \leq n\}$. [2]

(b) Let a be an algebraic number and r a rational number. Show that ra is an algebraic number. [5]

(c) Suppose you want to show that $A \Rightarrow B$ is **false**. How should you do this? What should you try to show about the truth of A and B? [2]

(d) Apply your answer of part (a) to show that the statement "If x is a real number that satisfies $-3x^2 + 2x + 8 = 0$, then $x > 0$" is false. [3]

(e) Write the negation of the statement: "The real-valued function f of one variable is *continuous at the point* x if and only if for every real number $\varepsilon > 0$, there is a real number $\delta > 0$ such that, for all real numbers y with $|x - y| < \delta$, $|f(x) - f(y)| < \varepsilon$." [5]

QUESTION 2 66

(a) Let P be the statement "All girls are good at mathematics." Which of the following statements is the negation of P?

(i) All girls are bad at mathematics;

(ii) All girls are not good at mathematics;

(iii) Some girl is bad in mathematics;

(iv) Some girl is not good at mathematics;

(v) All children who are good at mathematics are girls;

(vi) All children who are not good at mathematics are boys;

Can you find any statement in this list that has the same meaning as statement P? [4]

(b) Using truth tables, analyze the following argument and then state whether it is valid or invalid.

"It is not true that he is rich and arrogant. He is rich. Therefore he is not arrogant."

[8]

(c) Show that the polynomial $p(x) = x^4 - 2x^2 - 3$ has a root that lies between $x = 1$ and $x = 2$. [4]

(d) Prove by the *indirect uniqueness* method that if m and b are real numbers such that $m \neq 0$, then there is a unique real number z such that $mz + b = 0$. [4].

QUESTION 3

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(a) Suppose that Canada Post prints only 3 cent and 5 cent stamps. Prove that it is possible to make up any postage of n cents using only 3 cent and 5 cent stamps for $n \geq 8$. [8]

(b) (i) What is meant by a *square-free natural number*? [2]

(ii) Prove that the square root of any square-free natural number is irrational. [10]

QUESTION 4

(a) Prove that in any set of $n+1$ pairwise distinct integers, there must be two whose difference is divisible by n. [7]

(b) Prove, by the contrapositive method, that if no angle of a quadrilateral $RSTU$ is obtuse, then the quadrilateral $RSTU$ is a rectangle. [6]

(c) (i) Show that if r is a nonzero rational number, then $r\sqrt{7}$ is an irrational number. [4]

(ii) Using the result in part (a), or otherwise, show that $\sqrt{28}$ is irrational. [3]

QUESTION 5

(a) et a be an integer. Prove that if a^2 is divisible by 3, so is a. [6]

(b) Prove that $\sqrt{3}$ is irrational. [8]

(c) Prove that at a party of $n \geq 2$ people, there are at least two people who have the same number of friends at the party (where the relation of being friends is assumed not to be reflexive). [6]

END OF EXAMINATION