UNIVERSITY OF SWAZILAND

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FINAL EXAMINATIONS 2013/2014

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ALL QUESTIONS IN SECTION A. 3. ANSWER ANY THREE QUESTIONS IN SECTION B.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

SECTION A

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QUESTION 1

(a) Determine whether or not the points $(0, -2, -5)$, $(3, 4, 4)$ and $(2, 2, 1)$ lie on a straight line. [4]

(b) Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length. [7]

(c) A child pulls a wagon along level ground in straight line by exerting a force of 30 lbs on a handle that makes an angle of 30 deg with the horizontal. Find the work done in pulling the wagon 80 ft. [3]

(d) Find the two unit vectors that are perpendicular to the plane containing the points $P(3, -6, 4)$, $Q(2, 1, 1)$, and $R(5, 0, -2)$. [6]

QUESTION 2

(a) Given the function $\phi = \phi(x, y, z) = 2x^2 + 3y^2 + z^2$, which has continuous first partial derivatives, find $\nabla\phi$. [2]

(b) Let $\nabla\phi = 2xy\,\hat{\mathbf{i}} + (x^2 + 2yz)\,\hat{\mathbf{j}} + (y^2 + 1)\,\hat{\mathbf{k}}$. Find $\phi = \phi(x, y, z)$ if $\phi(1, -2, 2) = 4$. [8]

(c) For each of the following surfaces, find unit vectors that are normal to the surface at the given point:

(i) $x^2 + y^2 + z^2 = 9$, at $P(0, 3, 0)$; [5]

(ii) $ax + by + cz = d$, at any point $P(x, y, z)$; [3]

(iii) $3x - 6y - 2z = 15$, at $P\left(10, 0, \frac{15}{2}\right)$. [2]

SECTION B

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QUESTION 3

(a) In cylindrical coordinates (s, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{r}(s, \theta, z) = s\cos\theta\hat{\mathbf{i}} + s\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}}.$$

Find:

(i) $\hat{s}$; [2]

(ii) $\hat{\theta}$; [2]

(iii) $\hat{z}$; [2]

(iv) the velocity vector $\mathbf{v}$; [2]

(v) $\dot{\hat{s}}$; [2]

(vi) $\dot{\hat{\theta}}$; [2]

(vii) $\dot{\hat{z}}$; and [1]

(viii) the acceleration vector [2]

for any particle moving in this coordinate system.

(b) Prove that if $\mathbf{v}$ is any vector of constant length, then $\mathbf{v}$ and $\frac{d\mathbf{v}}{dt}$ are orthogonal. [3]

(c) If $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$, prove that $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$. [2]

QUESTION 4

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If

$$\mathbf{r}(s) = a\cos\left(\frac{s}{\omega}\right)\mathbf{i} + a\sin\left(\frac{s}{\omega}\right)\mathbf{j} + b\frac{s}{\omega}\hat{\mathbf{k}}$$

where s denotes arc length and a, b and ω are constants, find:

(a) the unit tangent vector $\hat{\mathbf{T}}$; [7]

(b) the curvature κ; [4]

(c) the unit principal normal $\hat{\mathbf{N}}$; [3]

(d) the unit binormal vector $\hat{\mathbf{B}}$. [6]

QUESTION 5

(a) An inductor of 2 henries, a resistor of 4 ohms, and a capacitor of 0.05 farads are connected in series with a battery of $E = 100$ volts. At $t \leq 0$ the charge on the capacitor and the current in the circuit are zero. Find the charge and current at any time $t > 0$. [8]

(b) Solve the problem in (a) if now the battery is of e.m.f. $E = 100\sin(4t)$. [12]

QUESTION 6

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(a) A train takes time T to perform a journey. It travels for time $\frac{T}{n}$ with uniform acceleration, then for time $(n-2)\frac{T}{n}$ with uniform speed V, and finally for time $\frac{T}{n}$ with constant retardation. Prove that its average speed is

$$(n-1)\frac{V}{n}.$$

If the length of this journey is 64 km, the time taken on the whole journey is 60 minutes, and the uniform speed is 96 km/h, find the time which is occupied in traveling with the uniform speed. [6]

(b) Particle A, initially at rest, is projected from the origin with acceleration $\frac{\sqrt{3}}{2}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{j}}$. Particle B, at rest at the point $(\sqrt{3}, 0)$, is projected at the same instant with acceleration $\frac{1}{2}\hat{\mathbf{j}}$. Show that the particles collide and that the time of collision is $t = 2$. [7]

(c) A particle moving in a straight line is acted upon by a retarding force of kv^3 per unit mass, where k is a constant and v is the speed. Show that after traveling a distance x, the speed and time taken are given by

$$v = \frac{u}{1 + kux} \quad \text{and} \quad t = \frac{1}{2}kx^2 + \frac{x}{u},$$

where u is the initial speed. [7]

QUESTION 7

(a) Particles P and Q of mass $20\,g$ and $40\,g$, respectively, are simultaneously projected from points A and B on a horizontal ground. The initial velocity, $\mathbf{v}_{0P}$, of P makes an angle of $45°$ with the horizontal AB, and the initial velocity, $\mathbf{v}_{0Q}$, of Q makes an angle of $135°$ with the horizontal AB. Each particle has initial speed $49\,m/s$, and the separation AB is $245\,m$ long. Both particles are assumed to travel in the same vertical plane and assumed to collide after time T. After collision, P retraces its path whilst Q falls vertically to the ground.

(i) Determine the position of Q when it hits the ground. [4]

(ii) How much time after collision does the particle Q take to reach the ground? Take $g = 9.8\,m/s^2$. [7]

(b) A particle of unit mass moves subject to a central force. Determine the law of force if the path followed by the particle is a circular orbit through the origin. [9]

END OF EXAMINATION