University of Swaziland

80

Final Examination, December 2013

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Numerical Analysis I

Course Code : M311

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
   Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
   Answer ANY THREE questions.
   Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

1.1. Convert the following binary numbers

(a) $(0.1111\cdots)_2$, with n ones. [3]

(b) $(0.\overline{10})_2$. [3]

to their decimal equivalent.

1.2. Given the function $f(x) = 3 - \sqrt{9 - x}$

(a) Find a suitable $g(x)$ that has been reformulated to be algebraically equivalent to $f(x)$, with the aim of avoiding loss of significance error. [3]

(b) Compare the results of calculating $f(0.0001)$ and $g(0.0001)$ with six digits and chopping. [3]

1.3. Find the divided differences for the following data

x_i	1	$\frac{1}{2}$	3
$f(x_i)$	3	−10	2

[3]

1.4. Determine the machine representation in single precision on a 32 bit word length computer for the decimal number −285.75 [8]

1.5. Complete the following table

i	x_i	$P_i(0.5)$	$P_{i,i+1}(0.5)$	$P_{i,i+i,i+2}(0.5)$
0	0	0		
			3.5	
1	0.4	0.8		$\frac{27}{7}$
			?	
2	0.7	?		

[6]

1.6. Given a continuous function $f(x)$ with a root x^* in $[a, b]$

(a) Give the algorithm for the bisection method to estimate the root to within an error ε [6]

(b) Using the algorithm in part 1.6(a) solve the equation $x^2 - 7$. Perform 4 iterations given $a = 2.5$ and $b = 3$ [5]

SECTION B: ANSWER ANY 3 QUESTIONS 82

2. Let $f(x) = x^3 + x + 1$, $a = -1$, $b = 1$ and $x_0 = 0$.

   (a) Show that the iterations generated by Newton's method for solving $f(x) = 0$ converges on $[a, b]$. [6]

   (b) Show that the Newton's iterative formula for solving $f(x) = 0$ is given by

   $$x_{n+1} = \frac{2x_n^3 - 1}{3x_n^2 + 1}.$$

   [3]

   (c) Perform 3 iterations of Newton's method. [3]

   (d) List all the floating point numbers that can be expressed in the form

   $$x = \pm(0.b_1b_2) \times 2^k, \qquad k, b_1, b_2 \in \{0, 1\}.$$

   [8]

3. Given the following 3 points

x_i	0	3	5
$f(x_i)$	1	6	7

   (a) Find the Lagrange interpolating polynomial $P_2(x)$. [8]

   (b) Use $P_2(x)$ to approximate $f'(1)$. [2]

   [illegible] the Lagrange interpolating polyno- [illegible] [10]

[illegible]

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ 2x_1 - 4x_2 + 6x_3 &= 4 \\ 3x_1 - 9x_2 - 3x_3 &= -9 \end{aligned}$$

[6]

(c) Find the step size required to evaluate the integral

$$\int_0^2 \ln(1 + x)dx$$

by the composite Trapezoidal rule with accuracy $\varepsilon = 5 \times 10^{-9}$. [6]

5. (a) Use the two point Gaussian quadrature rule 83

$$\int_{-1}^{1} f(x)dx \approx f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right)$$

to approximate the integral

$$\int_{0}^{1} x^2 e^{-x} dx.$$

[10]

(b) Use Neville's iterative scheme to find the interpolating polynomial for the following data

x_i	1	2	3
$f(x_i)$	3	−10	2

Hence approximate $f(2.5)$. [10]

6. (a) Find the Newton form of the interpolation for the following data

x	1	3	4	6
$f(x)$	−3	13	21	1

[8]

(b) Determine the interval width h so that the Simpson's rule can used to evaluate the integral

$$\int_{0}^{2} xe^{-x} dx$$

with an accuracy of 5×10^{-6}. [4]

(c) Solve the quadratic equation

$$x^2 + 62.10x + 1 = 0$$

as accurately as possible using 6 digits and rounding. [8]

END