University of Swaziland

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Supplementary Examination, July 2014

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Numerical Analysis I

Course Code : M311

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

1.1. Convert the following binary numbers

(a) $(1111\cdots)_2$, with n ones. [3]

(b) $(1.\overline{10})_2$. [3]

to their decimal equivalent.

1.2. Given the function $f(x) = x(\sqrt{x+1} - \sqrt{x}$

(a) Find a suitable $g(x)$ that has been reformulated to be algebraically equivalent to $f(x)$, with the aim of avoiding loss of significance error. [3]

(b) Compare the results of calculating $f(1000)$ and $g(1000)$ with six digits and chopping. [3]

1.3. Find the divided differences for the following data

x_i	1	$\frac{3}{2}$	0	2
$f(x_i)$	3	$\frac{13}{4}$	3	$\frac{5}{3}$

[3]

1.4. Determine the machine representation in single precision on a 32 bit word length computer for the decimal number 84.375 [8]

1.5. Complete the following table

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$
0	0	?		
			?	
1	0.4	?		$\frac{50}{7}$
			10	
2	0.7	6		

[6]

1.6. Given a continuous function $f(x)$ with a root x^* in $[a, b]$

(a) Give the algorithm for the bisection method to estimate the root to within an error ε [6]

(b) Using the algorithm in part 1.6(a) to find $\sqrt{10}$. Perform 3 iterations given $a = 3$ and $b = 3.5$ [5]

SECTION B: ANSWER ANY 3 QUESTIONS

2. Let $f(x) = x^3 - 2$, $a = 1$, $b = 2$ and $x_0 = 1$.

   (a) Show that the iterations generated by Newton's method for solving $f(x) = 0$ converges on $[a, b]$. [6]

   (b) Show that the Newton's iterative formula for solving $f(x) = 0$ is given by

   $$x_{n+1} = \frac{2(x_n^3 + 1)}{3x_n^2}$$

   [3]

   (c) Perform 3 iterations of Newton's method. [3]

   (d) List all the floating point numbers that can be expressed in the form

   $$x = (0.1b_1b_2b_3), \qquad b_1, b_2, b_3 \in \{0, 1\}.$$

   [8]

3. Given the following 3 points

x_i	-1	1	5
$f(x_i)$	3	-2	4

   (a) Find the Lagrange interpolating polynomial $P_2(x)$. [8]

   (b) Use $P_2(x)$ to approximate $f'(1)$. [2]

   (c) Construct a quadrature rule by using the Lagrange interpolating polynomials on the interval $[0, 4]$ using the nodes 0, 2, 3. [10]

4. (a) Using the LU factorization (use gaussian elimination), find the parabola $y = A + Bx + Cx^2$ that passes through the points $(1, 6)$, $(2, 5)$ and $(3, 2)$. [14]

   (b) Consider the linear system

   $$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 3x_3 & = & 6 \\ 2x_1 & - & 4x_2 & + & 6x_3 & = & 4 \\ 3x_1 & - & 9x_2 & - & 3x_3 & = & -9 \end{array}$$

   perform 2 iterations of the Gauss Seidel method with

   $$\underline{x}^{(0)} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}.$$

   [6]

5. (a) Use Neville's iterative scheme to find the interpolating polynomial for the following data

x_i	1	3	4
$f(x_i)$	-3	13	21

   Hence approximate $f(2.5)$. [10]

(b) Use the two point Gaussian quadrature rule

$$\int_{-1}^{1} f(x)dx \approx f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right)$$

to approximate the integral

$$\int_{0}^{2} xe^{-x}dx.$$

[10]

6. (a) Find the Newton form of the interpolation for the following data

x	-1	1	5	-3
$f(x)$	3	3	-2	4

[12]

(b) Solve the quadratic equation

$$x^2 - 102.4x + 1 = 0$$

as accurately as possible using 6 digits and rounding. [8]

END