UNIVERSITY OF SWAZILAND

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FINAL EXAMINATION, 2013/2014

B.Sc. III, BASS III, B.Ed III

Title of Paper : Complex Analysis

Course Number : M313

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked**.
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: Answer ALL Questions

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A1. (a) What is the principal part of $argz$?

(b) Show $Re(iz) = -Imz$.

(c) Define and give example of a closure of a set S of complex numbers (2,2,3)

A2. (a) Show that a linear transformation $\omega = Az + B$, where A and B are constant complex numbers, may be subdivided into a sequence of simple transformation. Describe them.

(b) Using Cauchy-Riemann equation (CRE)state the sufficient conditions theorem for a function $f(z)$ be differentiable at z_0 (5,5)

A3. Give definition and example of

(a) Analytic function.

(b) Singular point. (2,2)

A4. Prove that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then u and v are harmonic in D. (6)

A5. State

(a) Cauchy-Goursat theorem,

(b) Laurent series theorem. (3,3)

A6. State and prove the Residue theorem. (7)

SECTION B: Answer any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) Construct the line $Re\frac{1}{z} = \frac{1}{2}$. (4)

(b) Find and sketch the region into which the triangle $x = 1, y = x, y = 0$ is mapped by the transformation $w = 1 + z^2$. (4)

(c) Explain formular $\lim_{z \to z_0} f(z) = w_0$. (2)

(d) Find the limits. Give your reasonings

(i) $\lim_{z \to i} \frac{iz + 2i}{z - i}$,

(ii) $\lim_{z \to \infty} \frac{5z^2}{(z-1)^2}$. (3,2)

(e) Using just definition of derivative, find f' if

(i) $f(z) = (z + 1)^2$

(ii) $f(z) = Rez$. (2,3)

QUESTION B2 [*20 Marks*]

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B2. (a) (i) Prove that if $f(z) = u + iv$ is differentiable at z_0, then u and v satisfy CRE,
(ii) then show that $f(z) = x^2 + i2y$, where $z = x + iy$, is not differentiable. (5,2)

(b) Derive CRE in polar coordinates. (6)

(c) Find the analytic function $f(z)$, given that the real part $u = y^3 - 3x^2y$. (7)

QUESTION B3 [*20 Marks*]

B3. (a) Evaluate $\int_c \bar{z}dz$, where c, is the right-hand half of the circle $|z| = 2$, from $z = -2i$ to $z = 2i$. (6)

(b) Apply Cauchy integral formula to evaluate

(i) $\displaystyle\int_c \frac{z^2dz}{(z^2+1)(z^2+9)}$,

if $c = \left\{ z : |z| = 2 \quad \text{in positive direction, and} |z| = \frac{3}{2} \quad \text{in negative direction} \right\}$

(ii) $\displaystyle\int_c \frac{dz}{z^2+3z}$, where c is a positively oriented circle $|z| = 2$. (3,5)

(c) (i) State a Taylor series theorem,

(ii) and thus expland

$f(z) = \frac{1}{1-z}$ in Maclaurin series (2,4)

QUESTION B4 [*20 Marks*]

B4. (a) Expand in Laurant series

(i) $f(z) = \dfrac{e^z}{z^2}$, near $z_o = 0$,

(ii) $f(z) = \dfrac{1}{1+z}$, in powers of z in the domain $1 < |z| < \infty$. (3,4)

(b) Consider $f(z) = \dfrac{e^{-z}}{(z-1)^2}$.

(i) Find residue at $z = 1$,

(ii) and thus evaluate $\int_c f(z)dz$, where c is a positively oriented circle $|z| = 2$. (5,2)

(c) Consider $f(z) = \frac{1-e^{2z}}{z^4}$.

(i) Show that $z = 0$ is a pole. Find its order,

(ii) Find residue at $z = 0$. (3,3)

QUESTION B5 [*20 Marks*]

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B5. (a) Consider $f(z) = \frac{\sin z}{z}$.

(i) Show that $z = 0$ is removable singular point.

(ii) Let $g(z) = f(z)$ for $z \neq 0$.

Define $g(0)$ to make $g(z)$ entire function (4,2)

(b) Apply the Residue theorem to evaluate

(i) $\displaystyle\int_c \frac{dz}{z^3(z+4)}$, where c is a positively oriented circle $|z| = 2$.

(ii) $\displaystyle\int_0^\infty \frac{dx}{(x^2+1)^3}$ (6,8)

END OF EXAMINATION PAPER