University of Swaziland

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Final Examination, 2013/2014

B.Sc. III

Title of Paper : Dynamics II

Course Number : M355

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

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A1. Give definition and example of

a) generalized coordinates, (2)

b) non-holonomic system, (2)

c) scleronomic system, (2)

d) conservative system, (2)

d) virtual displacement. (2)

A2. A particle of mass m is hanged on a weightless spring of stiffness c.

a) Derive Lagrange equation, (2)

b) Solve it, (2)

c) Introduce generalized momentum, (2)

d) Construct Hamiltonian, (2)

e) Write Hamilton equations. (2)

A3. In polar coordinates (r, θ)

$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \Pi(r)$. Show that generalized momentum p_θ is conserved. (3)

A4. Check if the transformaton

$Q = q - 2, \quad P = p + 2q - 3$ is canonical. (3)

A5. Let A, B and C be arbitrary functions of q, p, t. Prove that

$[A, B + C] = [A, B] + [A, C]$ in the usual notations. (4)

A6. In the calculus of variations define boundary conditions

a) essential,

b) natural, (2,2)

A7. Find the extremals of

$$V[y(x)] = \int_0^{\frac{\pi}{2}} [(y')^2 - y^2]dx, \quad y(0) \text{ is free,}$$

$y(\frac{\pi}{2}) = 0.$ (4)

A8. For the functional

$$V[y(x)] = \int_{x_0}^{x_1} F(x, y(x), y'(x) \cdots, y^{(n)}(x))dx$$

write Euler-Poisson equation. (2)

SECTION B: ANSWER ANY *THREE* QUESTIONS

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QUESTION B1 [*20 Marks*]

B1. (a) Prove that the work done on a system is related to generalized forces and corresponding generalized coordinates by

$$Q_i = \frac{\partial W}{\partial q_i}, \quad i = \overline{1, n}. \qquad (5)$$

(b) Prove the Interchange of d and ∂ Lemma

$$\frac{d}{dt}\left(\frac{\partial \bar{r}_\nu}{\partial q_i}\right) = \frac{\partial \dot{\bar{r}}_\nu}{\partial q_i} \qquad (5)$$

(c) Two particles of masses m_1 and m_2 are conncected by a light inextensible string of length l and negligible mass which passes over a frictionless pulley. Set up the Lagrangian and find the acceleraion of mass m_1. (4)

(d) The Lagragian for a certain dynamical system is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}c(x^2 + y^2) - \frac{1}{2}c(y - x)^2.$$

Write down Lagrange's equations. (6)

QUESTION B2 [*20 Marks*]

B2. (a) Derive Lagrange's equations for the holonomic, scleronomic system with n degrees of freedom.

Hint: You may use results from question B1(b) and the cancellation of dot property lemma. (10)

(b) Derive Lagrange's equations for the double mathematical pendulum with the masses m_1 and m_2 and the strings of the length l_1 and l_2, respectively. (10)

QUESTION B3 [*20 Marks*]

B3. (a) Using just definition of Hamiltonian $H(q, p)$,

(i) derive Hamilton's equations and hence

(ii) Show that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$. (4,3)

(b) Consider a dynamic variable

$A(q, p, t)$, and let $H(q, p, t)$ be a Hamiltonian of a system.

(i) Prove that $\frac{dA}{dt} = \frac{\partial A}{\partial t} + [A, H]$,

and hence

(ii) Show that $A = q_1p_2 - q_2p_1$ is a constant of motion if

$$H = \frac{1}{2}(p_1^2 + p_2^2) + \frac{1}{2}\omega^2(q_1^2 + q_2^2). \qquad (3,4)$$

(c) Derive Hamilton equations in Poisson formulation. (6)

QUESTION B4 [*20 Marks*]

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B4. (a) Consider a functional

$$V[y(x)] = \int_{x_0}^{x_1} F(x, y(x), y'(x))dx,$$

subject to the boundary conditions $y(x_0) = y_0, y(x_1) = y_1$. Show that if $y(x)$ is an extremal,then it satisfies Euler eqution. (7)

(b) Find extremals of

$$V[(y(x)] = \int_{x_0}^{x_1} (y + xy')dx, \quad y(x_0) = y_0, y(x_1) = y_1.$$ (4)

(c) Let $F(y, y') = y\sqrt{1 + (y')^2}$. Construct

(i) Euler equation,

(ii) Beltrami identity. [5,4]

QUESTION B5 [*20 Marks*]

B5. (a) Find the extremals of

(i) $V[y(x), z(x)] = \int_0^1 [(y')^2 + (z')^2 + \cos x]dx$

$y(0) = 1, \quad z(0) = 2, \quad y(1) = 2, \quad z(1) = 1.$

(ii) $V[y(x)] = \int_{x_0}^{x} [2xy + (y''')^2]dx$ (6,8)

(b) Find Ostrogradski's equation for the following functional

$$V[z(x, y)] = \int\int_{\Delta} \left[\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} + 2g(x, y)z\right] dxdy,$$

where $z(x, y)$ is known on the boundary of region Δ. (6)

END OF EXAMINATION PAPER