UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, 2013/2014

B.Sc. III

Title of Paper : Dymanics II

Course Number : M355

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked**.
2. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1. Give definitions and examples of

(a) degree of freedon, (2)

(b) homonomic system, (2)

(c) rheonomic system, (2)

(d) non-conservative system, (2)

(e) generalized force. (2)

A.2 Consider a mathematical pendulum.

(a) Derive Lagrange equation (2)

(b) Solve it for small angle, (2)

(c) Introduce generalized momentum, (2)

(d) Construct Hamiltonian, (2)

(e) Write down Hamilton equations. (2)

A3. Prove that the generalized momentum conjugate to cyclic coordinate is conserved. (3)

A4. Check if the transformation $Q = q + 4, \quad P = p + q + 2$ is conanical. (3)

A5. (a) Define Poisson bracket between two physical quantities,

(b) Prove that for any two functions of q, p, t $[A, B] = -[B, A]$ (2,2)

A6. Derive formula for natural boundary condition. (4)

A7. Find extremals of

$V[y(x)] = \int_0^1 [(y')^2 + yy']dx,$

$y(0) = 2, \quad y(1)$ is free. (4)

A8. Consider functional

$$V[z(x, y)] = \int \int_{\Delta} F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})dxdy.$$

Write dowm Ostrogradski equation. (2)

SECTION B: ANSWER ANY *THREE* QUESTIONS

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QUESTION B1 [*20 Marks*]

B1. (a) Show that $\sum_{\nu=1}^{N} \overline{F_\nu} \cdot \delta \overline{r}_\nu = \sum_{i=1}^{n} Q_i \delta q_i$. in the usual notations. (5)

(b) Prove the cancellation of Dot property lemma, $\frac{\partial \dot{\overline{r}}_\nu}{\partial \dot{q}_i} = \frac{\partial \overline{r_\nu}}{\partial q_i}$. (5)

(c) Masses m_1 and m_2 are located on smooth inclined planes of fixed angles α_1 and α_2 respectively and are connected by an inextensibe string of negligible mass over a smooth peg. Set up the Lagrangian and find the acceleration of mass m_1. (4)

(d) The Lagrangian for a certain dynamical system is given by

$L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \frac{w}{2}(-\dot{x}y + \dot{y}x)$,

where w is a constant. Write down Lagrange's equations. (6)

QUESTION B2 [*20 Marks*]

B2. (a) Let the potential energy be $\Pi = \Pi(q, \dot{q})$. Show that $T + \Pi - \sum_{i=1}^{n} \dot{q}_i \frac{\partial \Pi}{\partial \dot{q}_i} = const$, in the usual notations (10)

(b) Two identical mathematical pendulums are hanged to the ceiling. The masses are connected with a spring of stiffnees c. Derive Lagrange's equations. (10)

QUESTION B3 [*20 Marks*]

B3. (a) Using just definitions of Hamiltonian $H(q, p, t)$,

(i) derive Hamilton equations and hence

(ii) show that for conservative system $H = T + \Pi$. (4,3)

(b) Let $A(q, p, t)$ be an arbitrary dynamic variable and $H(q, p, t)$ be a Hamiltonian of a sytem.

(i) Show that $\frac{dA}{dt} = \frac{\partial A}{\partial t} + [A, H]$,

(ii) and hence prove that $\omega q_1 \sin \omega r + p_1 \cos \omega t$ is a constant of motion if

$H = \frac{1}{2}(p_1^2 + p_2^2) + \frac{1}{2}\omega^2(q_1^2 + q_2^2)$, ω is a constant. (3,4)

(c) Apply Poisson bracket to show that transformation

$q = \lambda\sqrt{2Q} \cos P, \quad p = \frac{1}{\lambda}\sqrt{2Q} \sin P$ is canonical. (6)

QUESTION B4 [*20 Marks*]

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B4. (a) State and prove the Main Lemma of calculus of variations. (5)

(b) Consider a functional $V[y(x)] = \int_{x_0}^{x_1} F(y, y')dx$.

Derive Beltami identity. (6)

(c) Let $F(y, y') = y\sqrt{1-(y')^2}$. Construct

(i) Euler equation,

(ii) Beltrami identity. (5,4)

QUESTION B5 [*20 Marks*]

B5. (a) Find the extremals of

(i) $V[y(x), z(x)] \int_0^{\frac{\pi}{2}} [(y')^2 + (z')^2 + 2y'z']dx$

$y(0) = z(0) = 0, \quad y(\frac{\pi}{2}$ is free, $z(\frac{\pi}{2}) = 1$.

(ii) $V[y(x)] = \int_0^1 [(y'')^2 + y' + 3x^2]dx$

$y(0) = 0, \quad y(1) = y'(0) = y'(1) = 1.$ [6,8]

(b) Find Ostrogradski's equation for the following functional.

$$V[z(x, y)] = \int\int_\Delta [(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 + 2g(x, y)z]dxdy.$$

where $z(x, y)$ is known on the boundary of region Δ. (6)

END OF EXAMINATION PAPER