UNIVERSITY OF SWAZILAND

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FINAL EXAMINATION, 2013/2014

B.Sc. IV, BASS IV, B.Ed IV

Title of Paper : Fluid Dynamis

Course Number : M455

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

USEFUL FORMULAE

The gradient of a function $\psi(r,\theta,z)$ in cylindrical coordinates is

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{\partial\psi}{\partial z}\hat{k}$$

The divergence and curl of the vector field

$$\underline{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{k}$$

in cylindrical coordinates are

$$\nabla\cdot\underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\}$$

and

$$\nabla\times\underline{v} = \frac{1}{r}\det\begin{bmatrix}\hat{r} & r\hat{\theta} & \hat{k}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z}\\ v_r & rv_\theta & v_z\end{bmatrix}$$

The divergence of a vector

$$\underline{v} = v_r\hat{r} + v_\lambda\hat{\lambda} + v_\theta\hat{\theta}$$

in spherical coordinates

$$\nabla\cdot\underline{v} = \frac{1}{r^2}\frac{\partial(r^2v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial\lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta}$$

The convective derivative and Laplacian in cylindrical coordinates are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial\theta} + v_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

Identities

$$\underline{v}\cdot\nabla\underline{v} = \nabla\left(\frac{v^2}{2}\right) - \underline{v}\times\underline{\omega}$$

$$\nabla\times(\nabla\times\underline{a}) = \nabla\nabla\cdot\underline{a} - \nabla^2\underline{a}$$

SECTION A [*40 Marks*]: Answer ALL Questions

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A1. Define a continuum model on example of air density. (4)

A2. Describe the Lagrange method treating motion of continuum medium. (4)

A3. Write down a mass conservation equation in general case. (2)

A4. Prove $\overline{V} = \nabla\psi \times \overline{k}$ in the usual notations. (4)

A5. Consider the flow field represented by the stream function $\psi(x, y) = 10xy + 17$.
Is this flow
(a) Possible incompressible?
(b) irrotational? (3,3)

A6. Explain the term σ_{yy}. (3)

A7. Prove Archimedes' theorem. (5)

A8. Write down Euler equation for inviscid model in operator form. (2)

A9. Define
(a) Newtonian fluid;
(b) Similar flows. (4,2)

A10. Consider steady, incompressible inviscid flow. Given

$$\overline{V} \times \overline{\omega} = \nabla(\frac{V^2}{2} + \Phi + \frac{p}{\rho})$$

in the usual notations. Derive Bernoulli's equation. (4)

SECTION B: Answer any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) For the velocity field

$$\overline{V} = -ax\overline{i} + by\overline{j},$$

where a and b are positive constants, determine
(i) dimenstion of the flow;
(ii) whether the flow is steady;
(iii) equation for the streamline through point $(x, y) = (1, 1)$; put $a = b = 2$;
(iv) parametric equations for particle path located at $(x, y) = (2, 1)$ at $t = 0$. (1,1,4,4)
(b) Derive formular for convective derivative of density. (5)

(c) The component x of velocity in steady, incompressible flow field in the xy plane is $u = \frac{A}{x}$, where A is a constant.

Find the simplest y component of velocity for this flow field. (5)

QUESTION B2 [*20 Marks*]

B2. (a) Consider the velocity field

$$\overline{V} = Axy\bar{i} - \frac{1}{2}Ay^2\bar{j}$$

in the xy plane.

(i) Is this possible incompressible flow field?

(ii) Calculate the acceleration of a fluid particle at a point $(x, y) = (2, 1)$. (2,5)

(b) The vorticity of a certain incompressible flow is given by

$$\overline{\omega} = \begin{cases} -Ar\sin\theta, & \text{for } r < a \\ 0, & \text{for } r > a \end{cases}$$

Find the corresponding stream function. (8)

(c) Consider two-dimenstional incompressible flow. Derive general formula for stream function. (5)

QUESTION B3 [*20 Marks*]

B3. (a) Show that the pressure at a point of fluid is the same in all directions. (6)

(b) Consider uniformly rotating liquid with angular velocity ω in the field of gravity.

(i) Construct equilibrium equations.

(ii) Find pressure $p(r, z)$. (5,5)

(c) Write down the Navier-Stokes equations for compressible flow when density and viscosity are constants. (4)

QUESTION B4 [*20 Marks*]

B4. (a) The velocity distribution in a two-dimensional steady inviscid flow in the xy plane is

$$\overline{V} = (Ax - B)i + (C - Ay)\bar{j},$$

where $A = 2, B = 5$ and $C = 3$, body force $\overline{g} = -g\bar{k}$.

(i) Is flow incompressible?

(ii) Obtain an expression for the pressure gradient.

(iii) Evaluate the difference in pressure between point $(x, y, z) = (1, 3, 0)$ and the origin if the density is 1.2. (2,6,6)

(b) The velocity distribution for laminar flow between fixed parallel plates is given by

$$u = U\left[1 - \left(\frac{2y}{h}\right)^2\right]$$

where h is the distance separating the plates and the origin is placed midway between the plates. Consider $\mu = 1.1 \times 10^{-3}$ kg/ms, $U = 0.05m/s, h = 5mm$. Calculate

(i) the shear stress on the lower plate and give direction;

(ii) the force on a $0.3m^2$ section of the lower plate and give its direction. (4,2)

QUESTION B5 [*20 Marks*]

B5. (a) Consider steady viscous incompressible flow between two stationary plates located at $y = 0$ and $y = 1$, Given that pressure at $x = 0$ and $x = L$ is p_0 and p_L respectively, with $p_0 > p_L$. The effect of body forces is negligible.

(i) Put $\overline{V} = u(x, y)\bar{i}$ and simplify the Navier-Stokes equations to show that

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}.$$

(ii) Show that

$$p(x) = p_0 - \frac{p_o - p_L}{L} x.$$

(iii) Show that

$$u(y) = y(1 - y)\frac{p_o - p_L}{2\mu L}.$$

(5,3,3)

(b) Water flows steadily up a vertical 0.1m diameter pipe and out the mozzle, which is 0.05m in diameter, discharging to atmospheric pressure. The stream velocity at the nozzle exit must be 20 m/s. The flow is frictionless. The pipe is 4m long.

(i) Find velocity at inlet.

(ii) Caculate the gage pressure required at inlet. (3,6)

END OF EXAMINATION PAPER