University of Swaziland

Final Examination, December 2015

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) Define a critical number c, of a function $f(x)$. [3]

(ii) Find the critical number(s) for the function $h(x) = \sin^2 x + \cos x,\ 0 < x < 2\pi$. [4]

(iii) For $f(x) = -x^2 + 3x, \quad x \in [0, 3]$, determine whether the Rolle's Theorem can be applied to f. If yes, find all values of c in the open interval (a, b) such that $f'(c) = 0$. If not, explain why. [5]

(iv) Identify the open intervals on which the function $g(x) = x^2 - 2x - 8$ is increasing or decreasing. [3]

(b) Use appropriate rules to find the limits of the following functions.

(i) $\lim_{x \to \infty} \frac{5x^2}{x+3}$. [2]

(ii) $\lim_{x \to \infty} (\ln x)^{\frac{1}{x}}$. [6]

(c) (i) Write the first four terms of the sequence $a_n = (-1)^{n+1}\left(\frac{2}{n}\right)$. [4]

(ii) Find the limit of the sequence $a_n = \frac{2n}{\sqrt{n^2+1}}$. [2]

(d) (i) State the Root Test for series convergence. [4]

(ii) Use the Root Test to show that the series

$$\sum_{n=1}^{\infty} \left(\frac{1}{n+1}\right)^n$$

converges. [2]

(iii) Find the sum of the convergent series $\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$. [5]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Analyze and sketch a graph of the function $y = \dfrac{x^2}{x^2+3}$. Label any intercepts, relative extrema, points of inflection, and asymptotes. [12]

(b) For the following sequences with the given n^{th} term, determine if the sequence converges or diverges. If the sequence converges, find its limit.

(i) $a_n = (-1)^n \left(\dfrac{n}{n+1}\right)$. [4]

(ii) $a_n = \dfrac{10n^2+3n+7}{2n^2-6}$. [4]

Question 3

(a) State the definite integral representation for estimating the area of a region bounded by two curves. [4]

(b) Sketch the region bounded by the graphs of the following equations and find the area of the region.

$$f(y) = y^2, \quad g(y) = y + 2.$$

[8]

(c) State the alternating series test for convergence. [4]

(d) Use the alternating series test to show that the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

is convergent. [4]

Question 4

(a) Find the volume of the solid generated by revolving the region bounded by the following equations about the $x-$ axis.

$$y = x^2 + 1, \quad y = -x^2 + 2x + 5, \quad x = 0, \quad x = 3.$$

[10]

(b) Find the arc length of the graph of the function over the indicated interval.

$$x = \frac{1}{3}\left(y^2 + 2\right)^{\frac{3}{2}}, \quad 0 \leq y \leq 4.$$

[10]

Question 5

(a) Give the definitions of the n^{th} Taylor Polynomial and the n^{th} Maclaurin Polynomial. [4]

(b) Find the third Taylor polynomial, $P_3(x)$, for $f(x) = \sin x$, expanded about $c = \pi/6$. [6]

(c) Set up and evaluate the definite integral for the area of the surface generated by revolving the curve about the $x-$ axis

$$y = \sqrt{4 - x^2}, \quad -1 \leq x \leq 1.$$

[10]

Question 6

(a) Show that the geometric series $\displaystyle\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n$ converges and that its sum, $S = 6$. [4]

(b) (i) What is a monotonic sequence? [2]

(ii) Show that the sequence $a_n = \dfrac{n}{n^2 + 1}$ is monotonic. [6]

(c) (i) State the Second Derivative Test for local extrema. [4]

(ii) Find the local extrema for the function $f(x) = x^3 - 3x^2 + 3$. [4]