University of Swaziland

Supplementary Examination, July 2016

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) Define a point of inflection of a function $f(x)$. [2]

(ii) State the Second Derivative Test for Concavity. [3]

(iii) Determine the open intervals on which the graph of $f(x) = -x^3 + 6x^2 - 9x - 1$ is concave upward or concave downward. [4]

(iv) Explain why the Mean Value Theorem does not apply to the function $f(x) = \dfrac{1}{x-3}$ on $[0, 6]$. [2]

(b) Use appropriate rules to find the limits of the following functions.

(i) $\lim\limits_{x\to\infty} \dfrac{2x-1}{3x+2}$. [2]

(ii) $\lim\limits_{x\to 0^+} (\sqrt{x}\, \ln x)$. [6]

(c) (i) State the Ratio Test for series convergence. [4]

(ii) Use the Ratio Test to show that the series

$$\sum_{n=0}^{\infty} \left(\frac{n!}{3^n}\right)^n$$

diverges. [3]

(iii) Find the sum of the convergent series $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$. [5]

(d) (i) What is a power series? [3].

(ii) How do you test a power series for convergence? [3]

(iii) Define the Taylor series generated by a function $f(x)$ at a point $x = a$. [3]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Analyze and sketch a graph of the function $y = \frac{x}{\sqrt{x^2+2}}$. Label any intercepts, relative extrema, points of inflection, and asymptotes. [12]

(b) Determine the points of inflection and discuss the concavity of the graph of $f(x) = x^4 - 4x^3$ [8]

Question 3

(a) State the definite integral representation for estimating the area of a region bounded by two curves. [4]

(b) Sketch the region bounded by the graphs of the following equations and find the area of the region. $f(x) = \sqrt{x} + 3, \quad g(x) = \frac{1}{2}x + 3.$ [8]

(c) Use the definition of Taylor series to find the Taylor series, centered at $c = 1$ for the function $f(x) = \frac{1}{x}$. [8]

Question 4

(a) Find the volume of the solid generated by revolving the region bounded by the following equations about the $x-$ axis.

$$y = x^2, \quad y = 4x - x^2.$$

[10]

(b) Find the arc length of the graph of $y = \frac{x^3}{6} + \frac{1}{2x}$ over $\left[\frac{1}{2}, 2\right]$. [10]

Question 5

(a) Give the definitions of the n^{th} Taylor Polynomial and the n^{th} Maclaurin Polynomial. [4]

(b) (i) What is a monotonic sequence? [2]

(ii) Show that the sequence $a_n = \dfrac{2n}{1+n}$ is monotonic. [4]

(c) Apply L'Hôpital's rule to show that the sequence $a_n = \left(\dfrac{n+1}{n-1}\right)^n$ converges to e^2. [10]

Question 6

(a) (i) State without proof the theorem for convergence of a geometric series. [2]

(ii) Analyze the convergence of the geometric series $\sum_{n=0}^{\infty} \dfrac{3}{2^n}$. If it converges, find its sum. [4]

(b) Show that the integral test is applicable to the series $\sum_{n=1}^{\infty} \dfrac{n}{n^2+1}$. Apply the integral test to show that the series diverges. [6]

(c) (i) Use the Ratio Test to find the radius of convergence of the power series $\sum_{n=0}^{\infty} 3(x-2)^n$. [4]

(ii) Give the definitions of a P-series and a harmonic series. [4]