University of Swaziland

Final Examination, May 2016

B. Sc., B. Ed., B. A. S. S., B. Eng

Title of Paper : Calculus II

Course Code : M212

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

QUESTION 1

a. Consider the polar curves given by the equations

i. $r = \tan\theta \sec\theta$, [3]

ii. $r = \dfrac{1}{3 - 3\cos\theta}$. [5]

Find their cartesian equations and describe or identify the curves.

b. Eliminate the parameter t to find a cartesian equation of the curve [2]

$$x = e^{2t}, \quad y = t + 1.$$

c. Define the continuity of a function $f(x, y)$ at the point (x_0, y_0). [3]

d. Find the limit, if it exists, or show that the limit doesn't exist

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^4 + y^4}$$

[4]

e. Verify that $z = \ln(e^x + e^y)$ is a solution of the differential equation

$$\frac{\partial^2 z}{\partial x^2}\left(\frac{\partial^2 z}{\partial y^2}\right) - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$$

[7]

f. Find the polar coordinates with $0 \leq \theta \leq 2\pi$, of the point with cartesian coordinates $(1, -\sqrt{3})$. [3]

g. Find the directional derivative of the function $f(x, y, z) = \ln(3x + 6y + 9z)$ at the point $(1, 1, 1)$ in the direction of the vector $\underline{v} = 4\hat{i} + 12\hat{j} + 6\hat{k}$. [5]

h. Evaluate $\displaystyle\iint_R (3x + 4y^2)\,dA$, where R is the region in the upper half plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. [8]

SECTION B: ANSWER ANY 3 QUESTIONS

QUESTION 2

a. Evaluate the iterated integral by first converting it to polar coordinates

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2y^2dxdy$$

[10]

b. Evaluate

$$\iiint_E xe^{x^2+y^2+z^2}dV$$

where E is the unit ball $x^2 + y^2 + z^2 \leq 1$. [10]

QUESTION 3

a. Given the polar curve $r = 1 - \cos\theta$, $0 \leq \theta \leq 2\pi$

i. Sketch the curve. [4]

ii. Find the length of the curve. [10]

b. Use the chain rule to find $\dfrac{dz}{dt}$ where

$$z = x^2 + y^2 + xy, \quad x = \sin t, \quad y = e^t.$$

[3]

c. Find the cartesian coordinates of the point with polar coordinates $(2, -\frac{7\pi}{6})$. [3]

QUESTION 4

a. Use the Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y) = 3x + 6y$$

subject to the constraint

$$x^2 + y^2 = 10.$$

[10]

b. Use a triple integral to find the volume of the tetrahedron T, bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$ and $z = 0$. [10]

QUESTION 5

a. Suppose z is defined implicitly as a function of x and y by the equation

$$x^2 - y^2 + z^2 - 2xyz = 4,$$

find

i. $\dfrac{\partial z}{\partial x}$, [3]

ii. $\dfrac{\partial z}{\partial y}$. [3]

b. Let $z = \arcsin(x - y)$, $x = s^2 + t^2$, $y = 1 - 2st$. Using the chain rule

i. find $\dfrac{\partial z}{\partial s}$. [5]

ii. show that $z_t = 2(2 - s^2 - t^2 - 2st)^{-0.5}$. [9]

QUESTION 6

a. Find and classify the critical points of

$$f(x, y) = 4xy - x^4 - y^4.$$

[10]

b. Evaluate

$$\iint_D (x + 2y)dA$$

where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$. [10]

END OF EXAMINATION