University of Swaziland

Supplementary Examination, July 2016

B. Sc., B. Ed., B. A. S. S., B. Eng

Title of Paper : Calculus II

Course Code : M212

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

QUESTION 1

a. Verify that the function $z = \ln(e^x + e^y)$, is a solution of the differential equations

i. $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} - 1 = 0,$ [4]

ii. $\dfrac{\partial^2 z}{\partial x^2}\dfrac{\partial^2 z}{\partial y^2} - \left(\dfrac{\partial^2 z}{\partial x \partial y}\right)^2 = 0.$ [6]

b. Use a double integral to find the area of the region bounded by the curves $xy = 2$, $x = 2\sqrt{2}$ and $y = 4$. [10]

c. Find and classify the critical points of the function $f(x, y) = x^4 + y^4 - 4xy + 1$. [10]

d. Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y) = x^2 + 2y^2$$

subject to the constraint

$$x^2 + y^2 = 1$$

[10]

SECTION B: ANSWER ANY 3 QUESTIONS

QUESTION 2

a. Consider the cardiod $r = 1 - \cos\theta$

i. Sketch the cardiod. [4]

ii. Find the length of the cardiod. [6]

b. Find an equation in polar coordinates for each of the following curves

i. $2x + 3y = 3$. [4]

ii. $x^2 - 2x + y^2 = 0$. [6]

QUESTION 3

Evaluate the following integrals

a.

$$\iint_D (6x^2 - 40y)\,dA,$$

where D is the triangle with vertices $(0,0)$, $(5,3)$ and $(0,3)$. [8]

b.

$$\iiint_E 2x\,dV$$

where E is the region under the plane $2x + 3y + z = 6$ that lies in the first octant.[12]

QUESTION 4

a. Suppose that $z = f(x,y)$, $x = r\cos\theta$ and $y = r\sin\theta$. Prove that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2$$

[14]

b. Find the directional derivative of the function $f(x,y,z) = x^3e^y + xz$ in the direction of the vector from $P_1(4,0,16)$ to $P_2(-2,1,4)$. [6]

QUESTION 5

a. Consider the curve $r = 1 - \sin\theta$

i. Sketch the curve. [4]

ii. Find the area enclosed by the curve. [8]

b. Find the volume under the surface $z = 16xy + 200$, above the region in the $x - y$ plane bounded by $y = x^2$ and $y = 8 - x^2$. [8]

QUESTION 6

a. Find the equation of the tangent plane to the surface $z = x\sin(x + y)$ at the point $(-1, 1, 0)$. [8]

b. Evaluate

$$\iint_R \frac{x}{\sqrt{x^2 + y^2}} dxdy$$

where R is the region bounded by the lines $y = x$, $y = -2$ and $x = 0$. [12]

END OF EXAMINATION