University of Swaziland

Final Examination, May 2016

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Ordinary Differential Equations

Course Code : M213

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.

2. Each question in Section B is worth 20%.

3. Show all your working.

4. Non programmable calculators may be used (unless otherwise stated).

5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) Find the differential equation of the family of curves

$$y = A\cos 2x + B\sin 2x$$

where A and B are arbitrary constants. [4]

(ii) What is the order of the differential equation whose solution is the circle

$$(x-a)^2 + y^2 = a^2, \text{ where } a \text{ is an arbitrary constant?}$$

[4]

(iii) Solve

$$\frac{dy}{dx} = xy + x + y + 1.$$

[4]

(b) Show the differential equation

$$e^x(\cos y\, dx - \sin y\, dy) = 0, \quad y(0) = 0,$$

is exact and then solve. [6]

(c) Solve the Bernoulli equation

$$3y' + xy = xy^{-2}.$$

[6]

(d) Solve

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^x.$$

[6]

(e) Solve the Euler Cauchy equation

$$x^2y'' - 3xy' + 3y = 0.$$

[6]

(f) Classify the singular point(s) of the equation

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0, \text{ where n is constant.}$$

[4]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

Solve the following differential equations.

(a) $\dfrac{dy}{dx} = \dfrac{xy + y}{xy + x}$. [5]

(b) $\dfrac{dy}{dx} = \dfrac{2}{x + 2y - 3}$. [5]

(c) $x\dfrac{dy}{dx} + \dfrac{y^2}{x} = y$. [5]

(d) $(x+1)\dfrac{dy}{dx} - ny = e^x(x+1)^{n+1}$, where n is constant. [5]

Question 3

(a) Show that if $\left[\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)\Big/ N\right] = f(x)$ a function of x alone, then $e^{\int f(x)dx}$ is an integrating factor of $M(x,y)dx + N(x,y)dy = 0$. [10]

(b) Solve the system of equations

$$\frac{dx}{dt} - 7x + y = 0,$$
$$\frac{dy}{dt} - 2x - 5y = 0.$$

[10]

Question 4

Find two linearly independent solutions of the equation

$$2x^2y'' + xy' - (x^2+1)y = 0$$

using Frobenuius method (generalized series solution method). [20]

Question 5

(a) For the differential equation

$$y' = p(x)y^2 + q(x)y + r(x)$$

if the particular solution is $y = v(x)$, show that substituting $y = v(x) + \frac{1}{z}$ will reduce the given non linear differential equation in to a linear differential equation in z.

Find the general solution of

$$y' = 2xy^2 + (1 - 4x)y + 2x - 1$$

if the particular solution is $y = 1$. [10]

(b) Use the method of variation of parameters to find the solution of

$$y'' - 2y' - 3y = e^x.$$

[10]

Question 6

(a) Given $y = x$ is a solution of

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0,$$

find another linearly independent solution by reducing the order. [10]

(b) Solve the following differential equation using Laplace transform method

$$y'' - 2y - 8y = 0, \quad y(0) = 3, y'(0) = 6.$$

[10]

Table 1: Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}[f(t)]$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at} - e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at} - be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{d^n f}{dt^n}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$g(t) = \begin{cases} 0, & 0 \le t \le a; \\ f(t-a), & a < t. \end{cases}$	$e^{-as}F(s)$