University of Swaziland

Supplementary Examination, July 2016

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Ordinary Differential Equations

Course Code : M213

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) Show that $x^2 + 4y = 0$ is a solution of

$$\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0.$$

[3]

(b) Solve

$$\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0.$$

[3]

(c) Sow that the differential equation

$$e^y dx + (xe^y + 2y)dy = 0$$

is exact and then solve. [6]

(d) Show that the integrating factor of a linear first order equation

$$y' + P(x)y = Q(x)$$

is given by

$$e^{\int p(x)dx}$$

also find the general solution of the given differential equation

$$\frac{dy}{dx} = 4y + 2x - 2x^2.$$

[8]

(e) Solve the Bernoulli equation

$$2xyy' = y^2 - 2x^3, y(1) = 2.$$

[6]

(f) Solve

$$x^2y'' - 3xy' + 3y = 0.$$

[6]

(g) Solve

$y'' + y = \csc x$ using the method of variation of parameters.

[8]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

Solve the following differential equations

(a) $\dfrac{dy}{dx} = e^{x+y} + x^2 e^y$. [6]

(b) $(1 + x^2)\dfrac{dy}{dx} + 2xy = 4x^2$. [6]

(c) $\dfrac{dy}{dx} = \dfrac{y + x + 2}{y - x + 1}$. [8]

Question 3

(a) Solve the system of equations

$$\frac{d^2x}{dt^2} - 3x - 4y = 0,$$
$$\frac{dy^2}{dt^2} + x + y = 0.$$

[12]

(b) Solve

$$y''' - 2y'' + 4y' - 8y = 0.$$

[8]

Question 4

(a) Using the method of undetermined coefficients solve

$$y'' - 2y' + y = x^2 + e^{2x}.$$

[10]

(b) It is given that $y_1 = x$ and $y_2 = \frac{1}{x}$ are two linearly independent solutions of the associated homogeneous equation of

$$x^2y'' + xy' - y = x, \qquad x \neq 0.$$

Find a particular solution and the general solution of the equation. [10]

Question 5

(a) If $y_1(x), y_2(x), \cdots, y_n(x)$ are n linearly independent solutions of the linear homogenous differential equation

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots a_{n-1} \frac{dy}{dx} + a_n y = 0$$

where a_i constant, then there linear combination, $y = c_1 y_1(x) + c_2 y_2(x) + \cdots + c_n y_n(x)$, where $c_i, i = 1, 2, \cdots, n$ constants is also a solution of the homogenous equation. [10]

(b) Using the method of Laplace transforms solve

$$y'' - 5y' + 4y = e^{2t}, \quad y(0) = 19/12, \ \ y'(0) = 8/3.$$

[10]

Question 6

(a) Classify the singular points of the differential equation

$$2x(x-2)^2 y'' + 3xy' + (x-2)y = 0.$$

[5]

(b) Find a series solution, about $x = 0$, of the equation

$8x^2 y'' + 2xy' + y = 0$ by Frobenious method (generalized series solution method).

[15]

Table 1: Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}[f(t)]$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at} - e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at} - be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{d^n f}{dt^n}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$g(t) = \begin{cases} 0, & 0 \le t \le a; \\ f(t-a), & a < t. \end{cases}$	$e^{-as}F(s)$