UNIVERSITY OF SWAZILAND

FINAL EXAMINATION, 2015/2016

B.Sc. II, B.Ed II

Title of Paper : Mathematics for Scientists

Course Number : M215

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: Answer ALL Questions

A1. Find the angle between $\overline{u} = (1, -2, 4)$ and $\overline{v} = (1, 2, 4)$. (4)

A2. State

(a) Mean value theorem,

(b) L'Hospital's rule,

(c) Maclaurin's formula. (3,2,3)

A3. Find the partial derivatives of $f(x, y) = x^2y^5$ at $p(-1, 2)$. (3)

A4. State the equality of mixed derivatives theorem. (2)

A5. For the function $f(x, y, z) = xy + z^2 + x + y$, find

(a) gradient,

(b) stationary points, (2,2)

A6. Describe Lagrange's method for optimization problems: minimize $f(x)$, subject to $h(x) = 0$. (4)

A7. Compute the volume under the graph of $z = f(x, y) = x + 4y$ over the region $0 < x < 2, \quad 1 < y < 2$. (5)

A8. Consider a double integral $\int\int_D f(x, y)dxdy$. What transformations should be done to pass to polar coordinates? (3)

A9. Give definitions and examples of

(a) ODE with homogeneous coefficients,

(b) exact ODE.

(c) characteristic (auxiliary) equation

for linear ODE with constant coefficients (2,2,3)

SECTION B: Answer Any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) Does the point $R(1, 2)$ lie on the line through $P(0, 2)$ and $Q(3, 4)$? (5)

(b) Use vector product to find a unit vector perpendicular to the vectors $\overline{a} = (1, -3, 2)$ and $\overline{b} = (2, -1, 3)$. (5)

(c) Find the volume of parallelepiped spanned by the directed segments $\overline{OA}, \overline{OB}$ and $\overline{OC}$, if the coordinates of A, B and C are $(1, 0, 0), (1, 1, 0)$ and $(0, 0, 4)$ respectively. (5)

(d) If $f(x) = \sqrt{x^2 + 9}$, find all numbers in the interval $(0, 4)$ for which the mean value theorem is satisfied. (5)

QUESTION B2 [*20 Marks*]

B2. (a) Evaluate

(i) $\lim_{x\to 0} \dfrac{x - \sin x}{x^3}$,

(ii) $\lim_{x\to 0^+} xe^{\frac{1}{x}}$. (4,4)

(b) (i) Use the quadratic approximation to compute $\sqrt{1+x}$ for small $|x|$ and estimate the error,

(ii) In particular compute $\sqrt{1.02}$. (4,3)

(c) Find the third Taylor's polynomial of $f(x) = 1 + x^2 + 2x^3$ about $x_0 = 1$. (5)

QUESTION B3 [*20 Marks*]

B3. (a) Apply the chain rule to evaluate f'_u and f'_v if $f(x,y) = \exp(xy), x = u^2, y = uv$. (6)

(b) If $f(u,v,w) = u^2 - w^2$, what is df? (3)

(c) Find and classify all stationary points of $f(x,y) = x^3 + y^3 - 3x - 3y$. (6)

(d) Maximize $f(x,y) = x^2 - y^2$, subject to $2x + y = 5$. (5)

QUESTION B4 [*20 Marks*]

B4. (a) Evaluate the volume of the solid below the surface $z = x^2 + 2y$ and over the region R, where $R = \{(x,y) : 0 < x < 2, x^2 < y < 2x\}$. (7)

(b) Compute $\int\int_R (x^2+y^2)^{\frac{3}{2}} dxdy$ in polar coordinates if R is the region in the first quadrant bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, and the coordinate axes. (5)

(c) Evaluate $\int_0^1 \int_0^{\sqrt{4-x^2}} \int_{2x-y}^{2x+y} zdz \ dy \ dx$. (8)

QUESTION B5 [*20 Marks*]

B5. (a) Solve IVP $2ydx = 3xdy$, when $x = -2, y = 1$. (5)

(b) Solve ODE with homogeneous coefficients $3(3x^2 + y^2)dx - 2xydy = 0$ (5)

(c) Test the equation $(x+y)dx + (x-y)dy = 0$ for exactness and solve it. (5)

(d) Solve $4y''' + 4y'' + y' = 0$. (5)

END OF EXAMINATION PAPER