UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, 2015/2016

B.Sc. II, B.Eng. II, BASS II, BED. II

Title of Paper : Linear Algebra

Course Number : M220

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1. (a) Give the definition of each of the following;

(i) A vector space

(ii) An orthogonal matrix

(iii) A symmetix matrix

(iv) a skew-symmetric matrix (10

(b) Find the eigenvalues and the corresponding eigenvectors for $A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & -1 \\ 4 & -4 & 5 \end{pmatrix}$ (10

A2. (a) Determine whether the following mappings are linear transformations

(i) $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x + y - z, 2x + y)$ (5)

(ii) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (z + 1, y)$ (5)

(b) Prove that the set $B = \{x^2 + 1, x - 1, 2x + 2\}$ is a basis for the vector space $V = p_2(x)$ (10

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [*20 Marks*]

(a) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be $T(x, y, z) = (x + y + z, x + 2y + 3z)$

(i) Find the standard matrix of T. (5)

(ii) Find the matrix of T relative to the R-bases
$B_1 = \{(1, 1, 0), (0, 1, 1), (0, 0, 1)\}$ $B_2 = \{(1, 2), (1, 3)\}$ (5)

(b) Verify the Cayley-Hamilton theorem for (5)

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$ (10

QUESTION B4 [*20 Marks*]

(a) Find the inverse of the matrix A in two ways

(i) using the augemented matrix $[A : I]$ (6)

(ii) by computing a product $E_k E_{k-1} \cdots E_2 E_1$ of elementary matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -2 \end{pmatrix}$$ (6)

(b) Prove that if A and B are both non singular $n \times n$ matrices, then the product AB is also non singular and $(AB)^{-1} = B^{-1}A^{-1}$ (4)

(c) Prove that $[M_i(\alpha)]^{-1} = M_i(\frac{1}{\alpha})$ $\alpha \neq 0$ (4)

QUESTION B5 [*20 Marks*]

(a) Solve the system

$$\begin{aligned} 2x_1 + 5x_2 - 8x_3 + 6x_4 &= 4 \\ x_1 + 2x_2 - 3x_3 + 4x_4 &= 1 \\ x_1 + 4x_2 - 7x_3 + 2x_4 &= 8 \end{aligned}$$

(8)

(b) For which k does the following system have non-trivial solutions

$$\begin{aligned} kx_1 + 2x_2 - x_3 &= 0 \\ (k+1)x_1 + dx_2 + ox_3 &= 0 \\ -x_1 + kx_2 + kx_3 &= 0 \end{aligned}$$

(8)

(c) Determine whether the vectors are linearly independent
$(2, 4, 0, 4, 3), (1, 2, -1, 3, 1), (-1, -2, 5, -7, 1)$ (4)

QUESTION B6 [*20 Marks*]

(a) (i) Give the definition of a basis of a vector space
(ii) Determine whether the vectors $(1, 1, 1), (1, 2, 3)$ and $(2, -1, 1)$ form a basis for $\mathbb{R}^3$ (8)

(b) Use the adjoint of A to find A^{-1} where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$ (7)

(c) Show that $\begin{pmatrix} 0 & 0 & 5 \\ 0 & 0 & -1 \\ -5 & 1 & 0 \end{pmatrix}$ is skew symmetric (5)

QUESTION B7 [*20 Marks*]

(a) Let V be a vector space, S_1 and S_2 be finite sets of non-zero vector in V such that $S_1 \subset S_2$. Show that

(i) S_1 linearly dependent $\Rightarrow S_2$ is also linearly independent (5

(ii) S_2 linearly independent $\Rightarrow S_1$ is also linearly independent. (5

(b) Prove that if a homogeneous system has more unknowns than the number of equations than it always has a nontrivial solution. (1(

END OF EXAMINATION PAPER