UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2015/2016

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	FOUNDATIONS OF MATHEMATICS
COURSE NUMBER	:	M231
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ALL QUESTIONS IN SECTION A. 3. ANSWER ANY THREE QUESTIONS IN SECTION B.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

SECTION A

QUESTION 1

Define the following:

(a) Proposition; [2]

(b) Hypothesis; [2]

(c) Logical equivalence; [2]

(d) Premiss; [2]

(e) Deductive reasoning; [2]

(f) Inductive reasoning; [2]

(g) Sound argument; [2]

(h) Formal fallacy; [2]

(i) Circular reasoning; [2]

(j) Counter example. [2]

QUESTION 2

(a) Using truth-tables, analyze the following argument and state whether it is valid or invalid:

> Rock festivals are designed for adolescents. Nothing designed for adolescents has any cultural value. Therefore rock festivals have no cultural value.

[10]

(b) Translate each of the following statements into logical expressions using predicates (or statements involving variables), quantifiers and logical connectives:

(i) Everyone is your friend and is perfect; [3]

(ii) One of your friends is perfect. [2]

(c) Express the statement

> There is no one in this class who knows French and Russian

using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the words "It is not the case that.") [5]

SECTION B

QUESTION 3

(a) Write down symbolically, the negation of each of the following statements:

(i) $\exists x \in \mathcal{U}$; $(\neg P(x) \vee Q(x))$; [4]

(ii) $\forall x \in \mathbb{R}\, \forall y \in \mathbb{R}\, \exists z \in \mathbb{R}$; $x^2 + y^2 < z$. [6]

(b) Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$, where $A \subseteq \mathbb{R}$. Determine the truth set of

$$\forall y \in A\, \exists x \in \mathbb{R};\ x + y < 5.$$

[5]

(c) Determine the truth value in $\mathbb{R}$ of:

(i) $\exists x \in \mathbb{R}$ such that $|x| = x$; [2]

(ii) $\exists x \in \mathbb{R} : 3x - 4 = 3x$. [3]

QUESTION 4

(a) Prove that there are infinitely many primes of the form $4k + 3$, where k is a nonnegative integer. [11]

(b) For each of the arguments below, explain which rules of inference are used at each step.

(i) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job." [3]

(ii) "Somebody in this class enjoys whale watching. Everybody who enjoys whale watching cares about ocean pollution. Therefore, there is someone in this class who cares about ocean pollution." [3]

(c) What is wrong with this argument? Let $H(x)$ be the statement "x is happy." Given the premise $\exists x, H(x)$, we conclude that $H(Lola)$. Therefore, Lola is happy. [3]

QUESTION 5

(a) Prove that the following statements are false:

(i) For all $n \in \mathbb{N}$, $n^2 - n + 87$ is a prime number.

(ii) For all $n \in \mathbb{N}$, $2n^2$ is an odd integer.

(iii) For some $n \in \mathbb{N}$, with $n \geq 2$, $n^2 + 2n$ is a prime integer. [6]

(b) Prove that if there are at least 6 people at a party, then either 3 of them knew each other before the party, or 3 of them were complete strangers before the party. [12]

(c) Show that the polynomial $p(x) = x^4 - 2x^2 - 3$ has a root that lies between $x = 1$ and $x = 2$. [4]

QUESTION 6

(a) Prove that in any set of $n + 1$ pairwise distinct integers, there must be two whose difference is divisible by n. [7]

(b) Prove, by the contrapositive method, that if no angle of a quadrilateral $RSTU$ is obtuse, then the quadrilateral $RSTU$ is a rectangle. [6]

(c) (i) Show that if r is a nonzero rational number, then $r\sqrt{7}$ is an irrational number. [4]

(ii) Using the result in (a), or otherwise, show that $\sqrt{28}$ is irrational. [3]

QUESTION 7

(a) Prove that the square root of any prime number is irrational. [10]

(b) Critic Ivor Smallbrain is watching the classic film $11.\overline{9}$ *Angry Men*. But he is bored and starts wondering idly exactly which rational numbers $\frac{m}{n}$ have decimal expressions ending in $0000\ldots$ (that is, ending in repeating zeros). He notices that this is the case if the denominator n is 2, 4, 5, 8, 10, or 16, and wonders if there is a simple general rule which tells us which rational numbers have this property.

Help Ivor by proving that a rational number $\frac{m}{n}$ (in its lowest terms) has a decimal expression ending in repeating zeros if and only if the denominator n is of the form $2^a 5^b$, where a and b are integers with $a, b \geq 0$. [10]

END OF EXAMINATION