UNIVERSITY OF SWAZILAND

MAIN EXAMINATION, 2015/2016

BASS II, BED II, BENG II, BSC II

Title of Paper : DYNAMICS I

Course Number : M255

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. A formula sheet is provided on the last page.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

(a) Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$, $\mathbf{b} = 2\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + \hat{\mathbf{k}}$, and $\mathbf{c} = 4\,\hat{\mathbf{j}} - 3\,\hat{\mathbf{k}}$. Find

i. $\mathbf{a}\cdot\mathbf{b}$, [

ii. $\mathbf{b}\times\mathbf{c}$, [

iii. a unit vector in the direction of $\mathbf{b}$, [

iv. the volume of the parallelepiped with sides $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. [

(b) Let $\mathbf{u} = 3xyz^2\,\hat{\mathbf{i}} + 2xy^3\,\hat{\mathbf{j}} - x^2yz\,\hat{\mathbf{k}}$. Find $\operatorname{div}\mathbf{u}$. [

(c) Let

$$\mathbf{r} = 3\cos 2t\,\hat{\mathbf{i}} + 3\sin 2t\,\hat{\mathbf{j}} + (8t-4)\,\hat{\mathbf{k}}$$

be the position vector of a particle at time t. Find

i. the velocity of the particle, [

ii. the speed of the particle, [

iii. the acceleration of the particle, [

iv. the unit tangent vector $\hat{\mathbf{T}}$, [

v. the curvature κ and the radius of curvature R, [

vi. the unit principal normal $\hat{\mathbf{N}}$, [

vii. the tangential component of acceleration, [

viii. the normal component of acceleration. [

(d) A train takes time T to perform a journey from rest to rest. It travels for time $\frac{T}{n}$ with uniform acceleration, then for time $(n-1)\frac{T}{n}$ with uniform speed V, and finally for time $\frac{T}{n}$ with constant deceleration. Show that the train's average speed is

$$(n-1)\frac{V}{n}.$$

[

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

In polar coordinates (ρ, θ), the position vector of an arbitrary point (x, y) is given by

$$\mathbf{r} = \rho\cos\theta\,\hat{\mathbf{i}} + \rho\sin\theta\,\hat{\mathbf{j}}.$$

Find

(a) $\hat{\rho}$ and $\hat{\theta}$,

(b) the position vector $\mathbf{r}$ in terms of $\hat{\rho}$ and $\hat{\theta}$,

(c) $\dot{\hat{\rho}}$ and $\dot{\hat{\theta}}$, the time derivatives of $\hat{\rho}$ and $\hat{\theta}$,

(d) the velocity $\mathbf{v}$ in terms of $\hat{\rho}$ and $\hat{\theta}$,

(e) the acceleration $\mathbf{a}$ in terms of $\hat{\rho}$ and $\hat{\theta}$.

QUESTION B3 [*20 Marks*]

(a) A car with initial speed u accelerates uniformly over a distance $2s$ which it covers in time t_1. It is then stopped by being retarded uniformly to rest over a distance s, which it covers in time t_2. Show that

$$\frac{u}{2s} = \frac{2}{t_1} - \frac{1}{t_2}.$$

(b) A particle of mass m is thrown vertically upwards with initial speed V. The air resistance at speed v is mkv^2, where k is a constant.

i. Show that the upward motion of the particle is given by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -kv^2 - g.$$

ii. Find an expression for $v(t)$.

iii. Find the time T to reach maximum height.

QUESTION B4 [*20 Marks*]

(a) Let $\phi(x,y,z)=\sqrt{x^2+y^2+z^2}$. Show that grad ϕ is the unit vector in the direction of $\mathbf{r}=x\,\hat{\mathbf{i}}+y\,\hat{\mathbf{j}}+z\,\hat{\mathbf{k}}$.

(b) Let $\phi:\mathbb{R}^3\to\mathbb{R}$ have continuous second order partial derivatives. Prove that $\operatorname{curl}(\operatorname{grad}\phi)=\mathbf{0}$.

(c) Let $\mathbf{F}(x,y,z)=2xy\,\hat{\mathbf{i}}+\left(x^2+2yz\right)\hat{\mathbf{j}}+\left(y^2+1\right)\hat{\mathbf{k}}$.

i. Verify that $\operatorname{curl}\mathbf{F}=\mathbf{0}$.
ii. Find $\phi:\mathbb{R}^3\to\mathbb{R}$ such that $\mathbf{F}=\operatorname{grad}\phi$.

QUESTION B5 [*20 Marks*]

(a) Particle A, initially at rest, is projected from the origin with acceleration $\mathbf{a}_A=\frac{\sqrt{3}}{2}\,\hat{\mathbf{i}}+\frac{1}{2}\,\hat{\mathbf{j}}$. At the same instant, particle B at rest at the point $(\sqrt{3},0)$, is projected with acceleration $\mathbf{a}_B=\frac{1}{2}\,\hat{\mathbf{j}}$. Show that the particles collide and find the time of collision.

(b) A projectile is fired with an initial speed of 200 m/s and an angle of elevation $60°$. Assuming $g=10\text{ m/s}^2$, find

i. the velocity vector of the projectile at any time t,
ii. the position vector of the projectile at any time t,
iii. the range of the projectile,
iv. The maximum height reached.

QUESTION B6 [*20 Marks*]

(a) Consider a particle with mass m, velocity vector $\mathbf{v}$ and position vector $\mathbf{r}$. Show that if the particle is moving under a central force, its angular momentum is conserved.

(b) Show that movement under a central force occurs in a plane which is perpendicular to the angular momentum $\mathbf{L}$.

(c) A body of mass m falls from rest from a height h above the ground. Show that it strikes the ground after a time $\sqrt{\dfrac{2h}{g}}$ with speed $\sqrt{2gh}$.

END OF EXAMINATION PAPER