UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, 2015/2016

BASS II, BED II, BENG II, BSC II

Title of Paper : DYNAMICS I

Course Number : M255

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. A formula sheet is provided on the last page.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

(a) Find λ so that the vectors $2\,\hat{\mathbf{i}}-4\,\hat{\mathbf{j}}+5\,\hat{\mathbf{k}}$ and $3\,\hat{\mathbf{i}}+\lambda\,\hat{\mathbf{j}}-2\,\hat{\mathbf{k}}$ are perpendicular. [4]

(b) Find the volume of the parallelepiped whose edges are the vectors

$$\mathbf{a}=2\,\hat{\mathbf{i}}+3\,\hat{\mathbf{j}}-\hat{\mathbf{k}},\quad \mathbf{b}=\hat{\mathbf{i}}-2\,\hat{\mathbf{j}}+2\,\hat{\mathbf{k}},\quad \mathbf{c}=3\,\hat{\mathbf{i}}-\hat{\mathbf{j}}-2\,\hat{\mathbf{k}}.$$

[4]

(c) Let $\mathbf{u}=3xyz^2\,\hat{\mathbf{i}}+2xy^3\,\hat{\mathbf{j}}-x^2yz\,\hat{\mathbf{k}}$. Find curl $\mathbf{u}$. [4]

(d) Let

$$\mathbf{r}=4\sin t\,\hat{\mathbf{i}}+4\cos t\,\hat{\mathbf{j}}+8\,\hat{\mathbf{k}}$$

be the position vector of a particle at time t. Find

i. the velocity of the particle, [2]
ii. the speed of the particle, [2]
iii. the acceleration of the particle, [2]
iv. the unit tangent vector $\hat{\mathbf{T}}$, [2]
v. the curvature κ and the radius of curvature R, [4]
vi. the unit principal normal $\hat{\mathbf{N}}$, [2]
vii. the tangential component of acceleration, [2]
viii. the normal component of acceleration. [2]

(d) A train takes time T to perform a journey from rest to rest. It accelerates uniformly from rest for a time pT, and retards uniformly to rest at the end of the journey for a time qT. During the intermediate time, it travels with uniform speed v. Show that the train's average speed is

$$\tfrac{1}{2}v(2-p-q).$$

[10]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

(a) Let $\phi(x,y,z) = 3x^2y - yz^2$. Find div (grad ϕ) at the point $(1,-1,1)$. [5]

(b) Let $\mathbf{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{\mathbf{k}}$ and let $r = |\mathbf{r}|$. Show that

$$\nabla\left(\frac{1}{r}\right) = -\frac{\mathbf{r}}{r^3}.$$

[5]

(c) Let $\nabla\phi = 2xy^2\,\hat{\imath} + \left(2x^2y + 2yz^2\right)\hat{\jmath} + 2y^2z\,\hat{\mathbf{k}}$. Find $\phi(x,y,z)$ if $\phi(2,1,-1) = 3$ [10]

QUESTION B3 [*20 Marks*]

(a) The acceleration of a particle is $\mathbf{a} = \hat{\imath} + 2\,\hat{\jmath}$. Its initial velocity is $\hat{\mathbf{k}}$ and its initial position is $\hat{\imath}$.

i. Find the velocity of the particle at any time t. [5]

ii. Find the position of the particle at any time t. [5]

(b) Two points are at a distance d apart. A particle starts from A and moves in the direction $\overrightarrow{AB}$ with initial velocity u and uniform acceleration a. At the same instant, a second particle starts from B and moves in the direction $\overrightarrow{BA}$ with initial velocity $2u$ and retardation a.

Show that the particles collide at time $\dfrac{d}{3u}$ after the beginning of motion and that if the particles collide before the second particle returns to B, then

$$ad < 12u.$$

[10]

QUESTION B4 [*20 Marks*]

(a) A particle is projected with speed V at an angle of elevation θ. Show that the horizontal distance $\bar{x}$ travelled is given by $\bar{x} = \dfrac{V^2}{g}\sin 2\theta$. [10]

(b) A body of mass m falls from rest from a height h above the ground. Show that it strikes the ground after a time $\sqrt{\dfrac{2h}{g}}$ with speed $\sqrt{2gh}$. [10]

QUESTION B5 [*20 Marks*]

(a) Particle A, initially at rest, is projected from the origin with acceleration $\mathbf{a}_A = \frac{\sqrt{3}}{2}\,\hat{\mathbf{i}} + \frac{1}{2}\,\hat{\mathbf{j}}$. At the same instant, particle B at rest at the point $(\sqrt{3}, 0)$, is projected with acceleration $\mathbf{a}_B = \frac{1}{2}\,\hat{\mathbf{j}}$. Show that the particles collide and find the time of collision. [8]

(b) A projectile is fired with an initial speed of 200 m/s and an angle of elevation 45°. Assuming $g = 10\ \text{m/s}^2$, find

i. the velocity vector of the projectile at any time t, [3]

ii. the position vector of the projectile at any time t, [3]

iii. the range of the projectile, [3]

iv. The maximum height reached. [3]

QUESTION B6 [*20 Marks*]

(a) Show that

$$x(t) = c_1 \cos\omega_0 t + c_2 \sin\omega_0 t$$

can be written as

$$x(t) = A\cos\left(\omega_0 t + \phi\right)$$

where $A = \sqrt{c_1^2 + c_2^2}$ and $\phi = \arctan\left(-c_2/c_1\right)$. [6]

(b) Find the amplitude and phase constant for the block-spring system with position given by

$$x(t) = \sqrt{3}\cos\omega_0 t - \sin\omega_0 t.$$

[4]

(c) A block of mass m is attached to a spring with spring constant k and is free to slide along a frictionless surface. At $t = 0$, the system is stretched at an amount $x_0 > 0$ from the equilibrium position and is released from rest.

Find the period of oscillation of the block and find the speed of the block when it first returns to the equilibrium position. [10]

END OF EXAMINATION PAPER