University of Swaziland

Final Examination, December 2015

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Numerical Analysis I

Course Number : M311

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) Derive the formula used with the Bisection method to find the minimum number of iterations, n to achieve a certain desired accuracy ϵ . [5]

(ii) Give two advantages of the Bisection method. [4]

(iii) Give the graphical derivation of the Newton-Raphson iteration scheme. [6]

(iv) Give one advantage of the Newton-Raphson method over the Bisection method. [1]

(b) (i) Determine the linear Lagrange interpolating polynomial that passes through the points $(2, 4)$ and $(5, 1)$. [6]

(ii) Given the data below, construct a table for the divided differences. Do not give the polynomial.

i	0	1	2
x_i	1.0	1.3	1.6
$f(x_i)$	0.7651977	0.6200860	0.4554022

[6]

(iii) Suppose $f(x) = \sin x$ is approximated by an interpolating polynomial $P_n(x)$ of degree 5 in $[0\ 1]$. Estimate the absolute error $|f(x) - P_n(x)|$ for any $x \in [0\ 1]$. [6]

(c) (i) How can the function, $f(x) = \dfrac{1}{\sqrt{x^2+1} - x}$, for large number like 10^8, be re-written to avoid problems due to loss of precision?. [2]

(ii) In applying the quadratic formula to find the roots of the equation

$$x^2 + 62.10x + 1 = 0,$$

which root will present problems of loss of significance? Support your answer. [4]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) List all the floating point numbers that can be expressed in the form

$$x = \pm(0.b_1b_2)_2 \times 2^k, \quad k, b_1, b_2, \in \{0, 1\}.$$

[8]

(b) Based on the above computer number set, give examples of numbers that may cause

(i) Overflow. [2]

(ii) Underflow. [2]

(c) Determine the machine representation in single precision on a 32-bit word length computer for -0.375. [8]

Question 3

Let $f(x) = x - e^{-x}$.

(a) Show that $f(x)$ has exactly one root in $[0, 1]$. [5]

(b) Compute an approximation to the root by taking four (4) steps of the **Bisection method.** [8]

(c) How many iterations would be required to locate this zero to a tolerance of 10^{-5}? [3]

(d) Compute an approximation to the root by taking three (3) steps of the **Newton's method** starting with $x_0 = 0.5$ [4]

Question 4

(a) Suppose that $f(-1) = 3$, $f(0) = 4$, and $f(2) = 5$. Find the Lagrange interpolating polynomial which interpolates these values, and use it to estimate $f'(0)$. [10]

(b) Consider the points $x_0 = 1$, $x_1 = 1.5$, and $x_2 = 2.5$ for a function $f(x)$. The divided differences are $f[x_2] = 5$, $f[x_1, x_2] = 15$ and $f[x_0, x_1, x_2] = 35$. Use this information to construct the complete divided differences table for the given points. [10]

Question 5

(a) Consider the integral $\int_0^{\pi} \sin x \, dx$. Suppose we wish to integrate it numerically with an error of magnitude less than 10^{-4}. Determine the values of h and n needed if we wish to use the composite **Trapezoidal rule?** [6]

(b) Evaluate $\int_0^2 e^{-2x} \, dx$ using the **Simpson's rule** with $n = 4$. [6]

(c) Find the coefficients a, b and c for the following three-point **Gaussian quadrature** rule;

$$\int_{-1}^{1} f(x) \, dx \approx af\left(-\sqrt{\frac{3}{5}}\right) + bf(0) + cf\left(\sqrt{\frac{3}{5}}\right).$$

[8]

Question 6

Consider the following system of equations.

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 3x_2 - x_3 &= 8 \\ - x_2 + 2x_3 &= -5 \end{aligned}$$

(a) Re-arrange the equations to derive the Jacobi iterative scheme. Find $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$, starting with the zero initial vector. [10]

(b) Repeat step (a) for the Gauss-Seidel iteration. [10]