University of Swaziland

Supplementary Examination, July 2016

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Numerical Analysis I

Course Number : M311

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) Convert 1678 to its binary equivalent. [3]

(ii) Determine the machine representation in single precision on a 32-bit word length computer (Marc-32) for 7712. [3]

(iii) How can the function $f(x) = e^x - x - 1$, near $x = 0$, be re-written to avoid problems due to loss of precision? [3]

(b) (i) Consider the equation $x - x^{1/3} - 2 = 0$. Given initial value $x_0 = 3$, use Newton method to find x_k, where $k = 1, 2, 3, 4$. [6]

(ii) Let $f(x) = x^2 - a$. Show that the Newton Method leads to the recurrence

$$x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n}).$$

[3]

(iii) Do three steps of the Secant Method for $f(x) = x^3 - 2$, using $x_0 = 0$ and $x_1 = 1$. [5]

(c) (i) What is polynomial interpolation? [2].

(ii) State the Theorem used to estimate the error for polynomial interpolation. [3]

(ii) Determine the linear Lagrange interpolating polynomial that passes through the points $(2, 4)$ and $(5, 1)$. [6]

(ii) Given the data below, construct a table for the divided differences. Do not give the polynomial.

i	0	1	2
x_i	1.0	1.3	1.6
$f(x_i)$	0.7651977	0.6200860	0.4554022

[6]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) In error analysis, what does loss of significance means? Describe **two** ways in which loss of significance can be avoided. Give examples to support your answer. [6]

(b) Determine the decimal number that has

$$0 - 10000100 - 01010101100000000000000$$

as its Marc-32 representation. [6]

(c) List all the floating point numbers that can be expressed in the form

$$x = \pm(0.b)_2 \times 2^{\pm k}, \quad k, b \in \{0, 1\}.$$

[8]

Question 3

Consider the function $f(x) = (x-2)^2 - \ln x$ on the interval $1 \leq x \leq 2$.

(a) Prove that there is exactly one root of this equation in this interval. [6]

(b) Use four(4) steps of the **Newton's method** with the initial guess $x_0 = 1.5$ to find a root of $f(x)$. [7]

(c) Use four(4) steps of the **Secant method** to approximate the zero of $f(x)$ using $x_0 = 1$ and $x_1 = 2$ as initial data. [7]

Question 4

(a) Suppose that $f(x) = \cos x$. Let $x_0 = 0, x_1 = 0.6$, and $x_2 = 0.9$.

(i) Construct the Lagrange interpolating polynomial of degree at most 2. [6]

(ii) Use the interpolating polynomial to approximate $f(0.45)$. [2]

(iii) Find the relative error. [2]

(b) For the following table construct the table of forward differences.

x	-1	-0.5	0	0.5	1
$f(x)$	4	2	2	4	8

[10]

Question 5

(a) Estimate the value of $\int_0^1 \frac{1}{1+x^2}\,dx$ by using the **Trapezoidal rule** with 3 points. [5]

(b) How many subintervals are needed to estimate $\int_0^1 e^{-x^2}\,dx$ using the **Trapezoidal rule** with error not exceeding 0.5×10^{-5}? [5]

(c) Construct a quadrature rule on [0 4] using nodes 0, 1, and 2. [10]

Question 6

(a) Consider the linear system of equations

$$\begin{aligned} 2x_1 - x_2 + 4x_3 &= 21 \\ 3x_1 + x_2 - 2x_3 &= -3 \\ x_1 + 4x_2 + x_3 &= 2. \end{aligned}$$

Calculate the first two iterates of the Gauss-Seidel method to solve the linear system, using the starting values $(x_1, x_2, x_3) = (0, 0, 0)$. [10]

(b) Repeat step (a) using the Jacobi method. [10]