UNIVERSITY OF SWAZILAND

FINAL EXAMINATION, 2015/2016

B.Sc./ B.Ed./ BASS III

Title of Paper : Dymanics II

Course Number : M355

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1. Give definition and example of

(a) Constraint, (2)

(b) generalized coordinate, (2)

(c) scleronomic system, (2)

(d) conservative system (2)

(e) transformation equation (2)

A.2 A particle of mass M is located on a smooth desk and is attached to a weightless spring of stiffness c.

(a) Derive Lagrange's equation, (2)

(b) Solve it, (2)

(c) Introduce generalized momentum, (2)

(d) Construct Hamiltonian, (2)

(e) Derive Hamilston's equations (2)

A3. For a certain system the kinetic energy T and potential energy Π are given by
$2T = ml^2(\dot{x}^2 + \dot{y}^2 \sin^2 x), \quad \Pi = -mgl\cos x.$

Show that coordinate y is cyclic. (3)

A4. Give at least one condition for canonical transformation. (3)

A5. Consider a dynamic variable $A(q,p,t)$ and let $H(q,p,t)$ be a Hamiltonian of a system. Prove that

$\frac{dA}{dt} = \frac{\partial A}{\partial t} + [A, H]$ in the usual notations. (4)

A6. Define,

(a) functional,

(b) variation of a function. (2,2)

A7. Find the extremals of

$$v[y,(x)] = \int_0^1 [(y')^2 + yy']dx, \qquad y(0) = 2, y(1) = 3.$$ (4)

A8. For the functional

$$v[y_1, y_2, \cdots, y_n] = \int_{x_0}^{x_1} F(x, y_1, y_2, \cdots, y_n, y_1', y_2', \cdots, y_n')dx$$

write Euler's equation. (2)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B1 [*20 Marks*]

B1. (a) Prove the Interchange of d and ∂ Lemma

$$\frac{d}{dt}\left(\frac{\partial \bar{r}_\nu}{\partial q_i}\right) = \frac{\partial \dot{\bar{r}}_\nu}{\partial q_i}. \qquad (6)$$

(b) Given Lagrange's equations (L.E.) in general case. Derive *L.E.* for conservative system. (4)

(c) Two particles of mass m_1 and m_2 are connected by a light inextensible string of length l which passes through a smooth hole in a smooth horizontal table. The mass m_2 lies on the table at a distance r from the hole. A gravitation field g acts on this system. If the mass m_1 moves only in vertical line, find the equation of motion for both masses. (4)

(d) The Lagrangian for a certain system is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}c(x^2 + y^2) - \frac{1}{2}c(y - x)^2.$$

Write down Lagrange's equations. (6)

QUESTION B2 [*20 Marks*]

B2. (a) Prove that for holonomic, scleronomic system the following formula is valid

$$\sum_{i=1}^{n} \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T. \qquad (10)$$

(b) Derive the equation of motion for the system made up of a mathematical pendulum for which the pivot point (of negligible mass) is free to move horizontally. (10)

QUESTION B3 [*20 Marks*]

B3. (a) Using just definition of Hamiltonian $H(q,p)$ derive Hamilton's equations. (4)

(b) The Lagrangian for a certain system is given in Quesiton B1(d). Find

(i) Generalized momenta,

(ii) Hamiltonian,

(iii) Hamilton's equations. (3,5,3)

(c) Derive Hamilton's equations in Poisson's formulation. (5)

QUESTION B4 [*20 Marks*]

B4. (a) Derive Euler's equation corresponding to the functional

$$v[y(x)] = \int_{x_0}^{x_1} F(x, y(x), y'(x))dx, \quad y(x_0) = y_0, y(x_1) = y_1. \qquad (7)$$

(b) Find extremals of

$$v[y(x)] = \int_0^{\frac{\pi}{2}} [(y')^2 - y^2]dx, \quad y(0) = 0, \quad y(\frac{\pi}{2}) \text{ is free} \tag{4}$$

(c) Let $F(y, y') = y\sqrt{1 + (y')^2}$. Construct

(i) Euler's equation,

(ii) Beltrami's identity. (5,4)

QUESTION B5 [*20 Marks*]

B5. (a) Find the extremals of

(i) $v[y(x), z(x)] = \int_0^1 (y'^2 + z'^2 + y'z')dx,$

$y(0) = z(0) = 0, \quad y(1) = 1, z(1) = 2.$

(ii) $v[y(x)] = \int_0^1 (y''^2 + 1)dx,$

$y(0) = 0, y'(0) = y(1) = y'(1) = 1.$ [7,7]

(b) Write down Ostrogradski's equation for the following functionals

(i) $v[z(x, y)] = \int\int_D F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})dxdy,$

(ii) $v[z(x, y)] = \int\int_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy.$ (3,3)

END OF EXAMINATION PAPER