UNIVERSITY OF SWAZILAND

FINAL EXAMINATION, 2015/2016

B.Sc. IV, BASS IV, B.Ed IV

Title of Paper : Fluid Dynamics

Course Number : M455

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

USEFUL FORMULAE

The gradient of a function $\psi(r,\theta,z)$ in cylindrical coordinates is

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{\partial\psi}{\partial z}\hat{k}$$

The divergence and curl of the vector field

$$\underline{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{k}$$

in cylindrical coordinates are

$$\nabla\cdot\underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\}$$

and

$$\nabla\times\underline{v} = \frac{1}{r}\det\begin{bmatrix}\hat{r} & r\hat{\theta} & \hat{k}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z}\\ v_r & rv_\theta & v_z\end{bmatrix}$$

The divergence of a vector

$$\underline{v} = v_r\hat{r} + v_\lambda\hat{\lambda} + v_\theta\hat{\theta}$$

in spherical coordinates

$$\nabla\cdot\underline{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial\lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta}$$

The convective derivative and Laplacian in cylindrical coordinates are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial\theta} + v_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

Identities

$$\underline{v}\cdot\nabla\underline{v} = \nabla\left(\frac{v^2}{2}\right) - \underline{v}\times\underline{\omega}$$

$$\nabla\times(\nabla\times\underline{a}) = \nabla\nabla\cdot\underline{a} - \nabla^2\underline{a}$$

SECTION A [*40 Marks*]: Answer ALL Questions

A1. Define

(a) Two-dimensional flow;

(b) steady flow;

(c) stagnation point;

(d) particle path. (1,1,1,1)

A2. Describe the Euler method treating motion of continuum medium. (4)

A3. Write down a formula for a Lagrangian acceleration. (2)

A4. Prove $\overline{V} = \nabla \times \psi\overline{k}$ in the usual notations. (4)

A5. A flow is represented by velocity

$$\overline{V} = 10x\overline{i} - 10y\overline{j} + 30\overline{k}.$$

Is this flow

(a) possible incompressible?

(b) irratational? (3,3)

A6. Explain the term τ_{xy} (3)

A7. Prove Archimedes theorem. (5)

A8. Write down Navier-Stakes equation for incompressible model of fluid. (3)

A9. (a) Define Reynolds number;

(b) How the notion of the similar flows is used in experiments? (2,3)

A10. Write down and comment on Bernoulli's equation. (4)

SECTION B: Answer any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) For the velocity field

$$\overline{V} = \frac{A}{x}\overline{i} + \frac{Ay}{x^2}\overline{j}$$

where A is a constant, determine

(i) dimension of the flow;

(ii) whether the flow is steady;

(iii) streamline through the point $(x, y) = (1, 3)$, and $A = 2$;

(iv) time required for the fluid particle to more from $x = 1$ to $x = 3$ in this flow field. (1,1,4,4

(b) Derive the formular for continuity equation (mass conservation). (5)

(c) A uniform flow field $\overline{V}$ is inclined at angle α above the x axis.

(i) Evaluate the velocity components u and v;

(ii) Determine the stream function for this flow field. (2,3)

QUESTION B2 [*20 Marks*]

B2. (a) Consider the velocity field

$$\overline{V} = xy^2\overline{i} - \frac{1}{3}y^3\overline{j} + xy\overline{k}$$

(i) If it is a possible incompressible flow field?

(ii) Calculate the acceleration of a fluid particle at a point $(x, y, z) = (1, 2, 3)$. (2,5)

(b) Consider the Rankine's Vortex

$$\overline{\omega} = \begin{cases} \omega\overline{k}, & \text{for } r < a \\ 0, & \text{for } r > a \end{cases}$$

(i) Find the stream function;

(ii) Find velocity V_θ (5,3)

(c) Consider a line vortex

$$v_r = 0, \quad v_\theta = \frac{c}{r}$$

(i) Find a stream function;

(ii) Find a circulation. (2,3)

QUESTION B3 [*20 Marks*]

B3. (a) Consider a fluid at rest in the field of gravity. Show that $\frac{dp}{dz} = -\rho g$ in the usual notations. (6)

(b) Find the equation of the upper surface of the rotating fluid in the field of gravity. (10)

(c) Define the Newtorian fluid. (4)

QUESTION B4 [*20 Marks*]

B4. (a) In a frictionless flow the velocity is

$$\overline{V} = Ax\overline{i} - Ay\overline{j}$$

where A is a constant, and body force $\overline{g} = -g\overline{k}$. The pressure at $(x, y, z) = (0, 0, 0)$ is $p_0 = 0$

(i) Is flow incopressible?

(ii) Construct the Euler equations

(iii) Find pressure $p(x, y, z)$. (2,6,6)

(b) The velocity distribution for laminar flow between fixed parallel plates is given by

$$u = U\left[1 - \left(\frac{2y}{h}\right)^2\right],$$

where h is the distance separating the plates and the origin is placed midway between the plates. Consider $\mu = 1.1 \times 10^{-3} kg/ms$, $U = 0.3m/s$, $h = 0.5mm$. Calculate

(i) the shear stress on the upper plate and give direction;

(ii) the force on a $0.5m^2$ section of the plate and give its direction. (4,2)

QUESTION B5 [*20 Marks*]

B5. (a) (i) Derive the Navier-Stokes equation in dimensionless form, introducing the characteristic length and velocity. Neglect body forces.

(ii) Find dimension of Reynold's number. (6,2)

(b) A u-tube acts as a water siphon. The bend in the tube is 1m above the water surface; the tube outlet is 7m below the water surface. The fluid issues from the bottom of siphon as a free jet at atmospheric pressure. If the flow is frictionsless, determine

(i) the speed of the free jet;

(ii) the absolute and the gage pressure of the fluid in the bend. (6,6)

END OF EXAMINATION PAPER