UNIVERSITY OF SWAZILAND

FINAL EXAMINATION, 2016/2017

B.Sc. II, B.Eng II, B.Ed II, BASS II

Title of Paper : Calculus II

Course Number : M212

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked**.
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: Answer ALL Questions

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A1. (a) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ for $f(x,y,z) = 3x^2y - \sin(\frac{2yz^3}{x^2})$ (6)

(b) Show that if $f(x,y) = x^2 + xy + y^2\sin(\frac{x}{y})$ then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f$ and $x^2f_{xx} + 2xyf_{xy} + y^2f_{yy} = 2f$ (8)

(c) Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ for

$$f(x,y) = x^4 - 4x^3y + 4xy^3 - y^4$$

(6)

A2. (a) For each of the following use a double integral to find the area bounded by the curves

(i) $y = x^2$ and $y = x^4$

(ii) $y = 4x + 8$ and $y = x^3 + 8$ (12)

(b) A rectangular garden is to be fenced on 3 sides using 1000 metres of fencing (the 4th side being a straight river's edge). Use Langrange multipliers to find the dimensions that would give the largest possible area. (8)

SECTION B: Answer any *THREE* Questions

QUESTION B1 [*20 Marks*]

(a) Find the critical points of the following function and test for relative maxima, minima and saddle points

$$f(x,y) = 4xy - x^4 - y^2$$

(10)

(b) Show that the ellipsoid $3x^2 + 4y^2 + 8z^2 - 24 = 0$ and the hyperboloid of two piece $4x^2 - 4y^2 - z^2 - 4 = 0$ are orthogonal (perpendicular) to each other at the common point $P_0(4\frac{\sqrt{5}}{5}, \sqrt{2}, 2\frac{\sqrt{5}}{5})$ (10)

QUESTION B2 [*20 Marks*]

(a) Show that the function

$$f(x,y) = \frac{xy}{x-y}$$

satisfies

$$x^2f_{xx} + 2xyF_{xy} + y^2f_{yy} = 0$$

(4)

(b) Evaluate the iterated integral

$$\int_0^8 \int_{+3\sqrt{y}}^2 e^{x^4} dx dy$$

(16)

QUESTION B3 [*20 Marks*]

(a) Express the given rectangular equations in polar

(i) $xy = 4$

(ii) $x^2 - 8x + y^2 + 7 = 0$ (4)

(b) Consider the curve

$$r = 2 + 2\sin\theta.$$

(i) Sketch the curve.

(ii) Find the area enclosed by the curve.

(iii) Find the length of the curve (16)

QUESTION B4 [*20 Marks*]

(a) Find the equation of the tangent to the surface

$f(x, y, z) = x^2 + 3y^2 - 4z^2 + 3xy - 10yz + 4x - 5z - 22$

at the point $(1, -2, 1)$. (10)

(b) Find the point on the place

$$x + 2y - 3z - 4 = 0$$

nearest to the origin. (10)

QUESTION B5 [*20 Marks*]

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(a) Evaluate the integral by converting to polar coordinates

$$\int_0^2 \int_{\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2 y^2 dx dy \tag{10}$$

(b) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-y^2-x^2}} x^3 yz dz dy dx \tag{10}$$

END OF EXAMINATION PAPER